Investment in Firm-Specific Skills and Efficient Labour Policies

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This paper investigates the characteristics of firm's investment in firm-specific skills and efficient labour policies under frictional labour markets. Workers with large general skills receive a large amount of firm-specific investment and generally face low unemployment rate because of high tightness of their market. There is, however, a possibility that the market tightness for such workers becomes low. In this case, firms invest so much in raising workers' skills and sustain highly productive matches, while they reduce their vacancies to avoid swelled cost. Efficient labour policy packages always contain a subsidy to the investment. Unemployment benefit harms firm's investment, but it can help to remedy the inefficiency of the market equilibrium. We finally obtain a modified version of the Hosios-Pissarides condition for optimal bargaining power.

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1 Intoroduction

The seminal work of Becker (1964) still gives labour market research a new direction to develop in. Recent analyses have been focusing on the role of investment in human capital, whether general or firm-specific, to account for the level of unemployment (Acemoglu (1999), Marimon and Zilibotti (1999) and Shintoyo (1999)). There are some studies to reestimate Becker's conclusion. Contrary to the seminal results, Acemoglu and Pischke (1999a,b) concluded that firms would pay for general training when labour markets are imperfect.

The scope of this paper is to examine the role of investment in firm-specific skills and to investigate labour policies to eliminate the inefficiency under frictional labour market. More specifically, this paper examines Shintoyo (1999)'s results from the social point of view. His model is successful for analysing the role of firm-specific investment, but does not consider a prescription to remove the inefficiency of the investment. We can not make investment in firm-specific skills in advance of matching. Since a decision on it must be conditional on being matched, the current status of the market such as its tightness do not directly affect the amount of the investment. It is one of the properties of firm-specific investment that have not been stressed in the traditional analyses.

The model in this paper has shown that workers with high ability will receive a large amount of investment in firm-specific skills, and their matches do not easily brake up. They are likely to be demanded in the market and face low unemployment rate. These results parallel those of Mortensen and Pissarides (1999a). There is, however, a possibility that the tightness of the market for such workers becomes low. In this case, firms invest so much in raising workers' skills and sustain highly productive matches, while they reduce their vacancies to avoid swelled cost.

Our analysis suggests that a stable relationship may sustain the inefficiency generated by one-sided investment decision, which is called hold-up, for a long time. Moen (1998) introduced contingent loans to avoid the hold-up problem. Sato and Sugiura (2003) examined

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labour policies such as subsidy to investment in general skills and unemployment benefit to improve the efficiency of the investment. We investigate the effects of such policies on the investment in firm-specific skills and explore efficient labour policy packages. To assess the social optimality, we consider the nature that a new match always accompanies the cost to raise firm-specific skills.

In general, subsidy to firm-specific skill acquisition raises firm's investment and reduces job destructions. However, it may cause adverse effects when the endogenous level of the investment is large. In this case, it may contribute to increasing job destructions, because it lowers the opportunity cost of hoarding the current match by reducing the investment cost incurred on the next hiring. This property is pecular to firm-specific investment, because it relies on the nature that the cost to raise firm-specific human capital is always a burden to a new match. An increase in unemployment benefit brings about negative effects on the equilibrium by reducing firm's investment in skills and increasing job destructions. The benefit harms firm's job creation, but it can help to remedy the inefficiency generated by the match externality. We derive a modified version of the Hosios-Pissarides condition for optimal bargaining power of workers.

This paper is constituted as follows. In Section 2, we introduce the basic structure of our model. Section 3 offers the social optimality conditions. In Section 4, we examine the effects of labour policies on the equilibrium and derive the conditions to induce efficient investment in firm-specific human capital. We conclude this paper in Section 5.

2 The Model

We consider an economy populated by firms and workers; both are risk neutral and have common discount rate ρ.[1)] Each firm has a single job and takes either of the following conditions, filled and producing or vacant and searching. Similarly, workers can be either unemployed and searching or employed and producing. When a worker searches a new job, he or she must enter the unemployment pool, that is, there is no on-the-job search.

The number of workers is normalised to one without loss of generality. At any moment, δ workers are born and enter the economy, while the same number of workers die and exit out of the labour market. The number of vacant jobs and unemployed workers are v and u respectively, and its ratio v/u are denoted by θ. Hereafter, we call it the tightness of the labour market. Normalisation allows the variable u to represent the unemployment rate. For notational simplicity, we define a modified discount rate r as $r \equiv \rho + \delta$.

At every moment, the number of new matches, denoted by m, is represented by the matching function $m = m(v, u)$, which is homogeneous of degree one. The probability that firms successfully find workers is shown as $q(\theta) \equiv m/v = m(1, u/v) = m(1, 1/\theta)$, while the probability that workers succeed in finding jobs is $\theta \cdot q(\theta) = m/u = m/v \cdot v/u$. We make the following standard assumptions; $q'(\theta) < 0$ and $q(\theta) + q'(\theta) \cdot \theta > 0$.

Each worker has accumulated his or her general skills before the entry to the labour market. Assuming that general skills are obtained only through school education, there are no more opportunities for its accumulation, though they are effective for a lifetime. In our model, the accumulated level of general skills, denoted by z, can be observed and the labour market is segmented by its difference.[2] When a match is successfully formed, a firm incurs the cost to make its partner accumulate firm-specific skills through training.[3] Firm-specific skills formed in a match are assumed to become perfectly valueless at the same time that the match terminates.

When a match is formed, the productivity realises its level $f(t, z) + \varepsilon$; worker's skill contribution $f(t, z)$ plus idiosyncratic shock ε. The former is a concave function with $f_1 > 0$, $f_{11} < 0$, while the complementarity between firm-specific and general skills may or may not exist, that is $f_{12} \leq 0$. The timing of shock variation follows Poisson process with arrival rate s. When there is change, the new value of ε is a drawing from the fixed distribution $G(\varepsilon)$, which has finite upper bound ε_u. The density function of the distribution is denoted by $g(\varepsilon)$.

Firms can create jobs that realise the upper support of the productivity distribution given worker's skill contribution.[4] Opened vacancies are always maintained at the best technology, since firms close the vacancies that are depreciated at no cost. Existing filled jobs are destroyed only if the idiosyncratic component of productivity falls below some critical number varepsilon $\varepsilon_d < \varepsilon_u$.

Vacancies cost k per unit time and firms pay the cost to raise firm-specific human capital $c(t) = ct$ when a match is formed. It is worth noting that firms are assumed to make lump-sum investment with the workers to simplify the analysis. This assumption can be justified as follows. Workers learn a lot about the customs of their workplaces and the characteristics of their jobs, which bring about the specificity of skills, in the first few years after joining the new firms. Then we obtain

$$\rho V = -k + q(\theta)\{J(t, z, \varepsilon_u) - ct - V\}, \quad (1)$$

where V and $J(\cdot)$ are respectively the asset values of a vacancy and a filled job. Free entry will drive the value of a vacancy down to zero, that is,

$$V = 0. \quad (2)$$

The value of a filled job to the firm solves

$$\rho J(t,z,\varepsilon) = f(t,z) + \varepsilon - w(t,z,\varepsilon) - \delta[J(t,z,\varepsilon) - V] + s\left[\int_{\varepsilon_d}^{\varepsilon_u} \{J(t,z,x) - J(t,z,\varepsilon)\}\, dG(x) + G(\varepsilon_d)\{V - J(t,z,\varepsilon)\}\right], \quad (3)$$

where $w(t, z, \varepsilon)$ is the wage paid to the worker and ε_d denotes a separation point of the match. The firm receives a payoff at each moment, while the match breaks up owing to worker's death with probability δ. The match will be either continued or terminated when a shock is hit with probability s.

Let $W(t, z, \varepsilon)$ be the asset value for an employed worker. Then we obtain

$$\rho W(t,z,\varepsilon) = w(t,z,\varepsilon) - \delta W(t,z,\varepsilon) + s\left[\int_{\varepsilon_d}^{\varepsilon_u} \{W(t,z,x) - W(t,z,\varepsilon)\}\, dG(x) + G(\varepsilon_d)\{U(z) - W(t,z,\varepsilon)\}\right] \quad (4)$$

where $U(z)$ denotes the value of being unemployed that is given by

$$\rho U(z) = b + q(\theta)\theta(W(t,z,\varepsilon_u) - U(z)) - \delta U(z). \quad (5)$$

We assume that the rent from a job match is shared between the firm and worker, according to the Nash bargaining solution. The Nash solution implies that

$$W(t,z,\varepsilon) - U(z) = \frac{\beta}{1-\beta}\{J(t,z,\varepsilon) - V\}, \quad (6)$$

where β is a parameter representing the bargaining power of workers. A job match breaks up when no positive rent is derived from it. Since the rent is monotonically increasing in ε, there is a unique reservation level of shock. When a realised value of shock is below the point, that is $\varepsilon < \varepsilon_d$, the match is terminated. The critical point is obtained by the equation

$W(t, z, \varepsilon_d) = U(z)$.

Using the equations from (3) to (6) except (5), we can obtain the wage equation as follows. (For details, see the Appendix A.)

$$w(t,z,\varepsilon) = \beta\left[f(t,z)+\varepsilon\right] + (1-\beta)rU(z). \quad (7)$$

We also have the asset value of the employed worker as follows. (For details, see the Appendix B.)

$$\begin{aligned} W(t,z,\varepsilon) &= \frac{1}{r+sG(\varepsilon_d)}\left[\beta f(t,z)\right. \\ &\quad \left. + \{(1-\beta)r + sG(\varepsilon_d)\}U(z)\right] \\ &\quad + \frac{\beta}{r+s}\left\{\frac{s}{r+sG(\varepsilon_d)}\int_{\varepsilon_d}^{\varepsilon_u} x dG(x) + \varepsilon\right\}. \end{aligned} \quad (8)$$

Evaluating the both sides of Eq. (8) by $\varepsilon = \varepsilon_d$ gives the asset value of the unemployed worker. Then we obtain

$$rU(z) = f(t,z) + \frac{r+sG(\varepsilon_d)}{r+s}\varepsilon_d + \frac{s}{r+s}\int_{\varepsilon_d}^{\varepsilon_u} x dG(x). \quad (9)$$

It is the condition that the asset value of the umemployed must satisfy at the equilibrium. Evaluating Eq. (8) by both $\varepsilon = \varepsilon_d$ and $\varepsilon = \varepsilon_d$, and taking the difference between them, we obtain

$$W(t,z,\varepsilon_u) - U(z) = \frac{\beta}{r+s}(\varepsilon_u - \varepsilon_d). \quad (10)$$

Using this relationship, Eq. (5) can be written as

$$rU(z) = b + q(\theta)\theta \cdot \frac{\beta}{r+s}(\varepsilon_u - \varepsilon_d). \quad (11)$$

We can obtain the following explicit expression for the asset value of a filled job (see the Appendix C).

$$\begin{aligned} J(t,z,\varepsilon_u) &= \frac{1-\beta}{r+sG(\varepsilon_d)}f(t,z) + \frac{(1-\beta)\varepsilon_u}{r+s} \\ &\quad + \frac{1-\beta}{r+sG(\varepsilon_d)}\left\{\frac{s}{r+s}\int_{\varepsilon_d}^{\varepsilon_u} x dG(x) - rU(z)\right\}. \end{aligned} \quad (12)$$

The firm chooses the optimal amount of investment in firm-specific skills by solving the following maximization problem.

$$\max_t J(t,z,\varepsilon_u) - ct$$

Then we obtain the first-order condition for the problem as follows.[5)]

$$\frac{1-\beta}{r+sG(\varepsilon_d)}f_1(t,z) = c. \quad (13)$$

Since a decision on firm-specific investment must be made conditional on being matched, the current status of the market such as its tightness do not directly affect the amount of the investment.

Firms post vacancies until the asset value reaches to zero. Utilising Eqs. (6) and (10), we obtain the following job creation condition from Eqs. (1) and (2).

$$\frac{1-\beta}{r+s}(\varepsilon_u - \varepsilon_d) - ct = \frac{k}{q(\theta)}. \quad (14)$$

Making use of Eqs. (9) and (11), we have the job destruction condition as follows.

$$\begin{aligned} f(t,z) + \frac{r+sG(\varepsilon_d)}{r+s}\varepsilon_d + \frac{s}{r+s}\int_{\varepsilon_d}^{\varepsilon_u} x dG(x) \\ = b + q(\theta)\theta \cdot \frac{\beta}{r+s}(\varepsilon_u - \varepsilon_d). \end{aligned} \quad (15)$$

The labour market equilibrium are characterised by the endogenous variables such as t, ε_d, and θ. These variables are determined by three equations from (13) to (15). Note that the level of general skills z is a given parameter. The flow of workers is shown in Figure 1. The unemployment rate is determined to equalise the inflow with the outflow of unemployment pool as follows.[6)]

$$u = \frac{sG(\varepsilon_d) + \delta}{\theta q(\theta) + sG(\varepsilon_d) + \delta}. \quad (16)$$

It follows that the unemployment rate increases with higher separation point and decreases with higher tightness.

The model in this paper contains a large number of variables to be analysed, but it possibly make the theoretical results obscure. To obtain a fine view of this model, we apply the comparative statics on endogenous variables t, ε_d, θ. Totally differentiating Eqs. (13), (14) and (15) gives

$$\begin{bmatrix} A_1 & A_2 & 0 \\ -c & \frac{-(1-\beta)}{r+s} & \frac{kq'}{q^2} \\ f_1 & B_2 & B_3 \end{bmatrix} \begin{bmatrix} dt \\ d\varepsilon_d \\ d\theta \end{bmatrix} = \begin{bmatrix} \frac{-(1-\beta)}{r+sG(\varepsilon_d)} f_{12} \\ 0 \\ -f_2 \end{bmatrix} dz, \quad (17)$$

with the following notations;

$$\begin{aligned} A_1 &\equiv \frac{1-\beta}{r+sG(\varepsilon_d)} f_{11} < 0, \\ A_2 &\equiv -\frac{(1-\beta)sg(\varepsilon_d)}{(r+sG(\varepsilon_d))^2} f_1 < 0, \\ B_2 &\equiv \frac{r+sG(\varepsilon_d)+\beta q(\theta)\theta}{r+s} > 0, \\ B_3 &\equiv -\frac{\beta}{r+s}(\varepsilon_u-\varepsilon_d)\left(q(\theta)+q'(\theta)\theta\right) < 0 \end{aligned}$$

Then, we obtain the following proposition.

Proposition 1 *Workers with a larger amount of general skills will receive a larger amount of investment in firm-specific skills and will face a smaller value of separation point. If determined amount of investment is not so large, the tightness of market for the workers will become stronger. If the investment is large enough, the tightness becomes weaker.*

Proof See the Appendix D.

As we treat z as exogenous, we can interpret that a higher value of z corresponds to highly educated or more able workers. We can derive the following implication from Proposition 1. Workers with high ablity will receive a large amount of investment in firm-specific skills, and their matches do not easily brake up. If firms do not invest so much in firm-specific skills, the market tightness for the workers is high. They will face low unemployment rate, because both lowered separation point and high tightness contribute tolowering the rate. The story in this case parallels Mortensen and Pissarides (1999a) and seems to be consistent with observed facts.

However, if firms invest so much in raising workers' skills, the market tightness will be lowered. In this case, they can enjoy highly productive matches for a long time, while the incurred cost is so large that firms reduce their vacancies to avoid the cost. The proposition says that highly developed training system inside the firms may cause a thin market for the workers. The complementarity between firm-specific and general skills has a significant effect to cause this case, if it exists.

3 Analysis of Optimality

In this section, we analyse whether the decentralised equilibrium is socially optimal or not. We compare it with those derived from the central planner's problem as is given in Pissarides (2000). Central planner controls θ, ε_d, and t to maximise the following social surplus.[7]

$$\Omega = \int_0^{\infty} e^{-\rho\tau}(y + bu - k\theta u - q(\theta)\theta u \cdot ct)d\tau. \quad (18)$$

Note that the last term in the bracket stands for the investment cost that is sunk when new matches are formed. The laws of motion of the state variables are given in the Appendix. Then we obtain the following necessary conditions. (For details, see the Appendix E.) For easy comparison with the decentralised economy, the social optimality conditions are differentiated by adding superscript asterisks.

$$\frac{s+\delta}{(r+s)(sG(\varepsilon_d^*)+\delta)} f_1(t^*, z) = c, \quad (19)$$

$$[1-\eta(\theta^*)]\left\{\frac{\varepsilon_u-\varepsilon_d^*}{r+s} - ct^*\right\} = \frac{k}{q(\theta^*)}, \quad (20)$$

$$f(t^*, z) + \frac{r+sG(\varepsilon_d^*)+q(\theta^*)\theta^*}{r+s}\varepsilon_d^* + \frac{s}{r+s}\int_{\varepsilon_d^*}^{\varepsilon_u} xdG(x) = b - k\theta^* - q(\theta^*)\theta^* ct^* + \frac{q(\theta^*)\theta^*\varepsilon_u}{r+s}. \quad (21)$$

Note that $\eta(\theta)$ denotes the elasticity of $q(\theta)$ with respect to θ, that is $\eta(\theta) \equiv -\frac{q'(\theta)\theta}{q(\theta)}$ Then we obtain the following proposition.

Proposition 2 *At the equilibrium of decentralised economy, investment in firm-specific skills is always socially small if* $\beta > \frac{s}{r+s}$ *holds.*

proof Comparing Eq. (13) with Eq. (19), we know that the decentralised economy attains socially underinvestment in firm-specific skills if the following inequity holds;

$$\frac{(s+\delta)(r+sG(\varepsilon_d))}{(r+s)(sG(\varepsilon_d^*)+\delta)} > 1-\beta. \tag{22}$$

Let us obtain the minimum value of the left hand side (LHS) at first. Evaluating LHS by both $G(\varepsilon_d)=0$ and $G(\varepsilon_d^*)=1$, the minimum value of LHS is proved to be $\frac{r}{r+s}$. Therefore, if $\frac{r}{r+s} > 1-\beta$, or equivalently, $\beta > \frac{s}{r+s}$ holds, LHS is always larger than the opposite side. Note that "if-statement" is one of sufficiency. (QED)

Proposition 2 states that a small rate of separation is likely to generate a hold-up problem. It may be against our intuition that low separation rate corresponds to a stable relationship between worker and firm and leads to efficient investment in skills. Theoretically speaking, the stable relationship sustains the inefficiency generated by one-sided investment decision for a long time. If we think of stable relationship as a source of social efficiency, we should consider the possibility that strong ties between a worker and firm brings a cooperative decision over investment in skills. As our model contains an exogenous structure of investment decision, it may generate a somewhat paradoxical result.

We show the following lemma to understand our model.

Lemma 1 *The decentralised economy attains socially underinvestment in firm-specific skills if a separation point coincides with that of the social optimum.*

Proof Let us evaluate LHS of Eq. (22) at $\varepsilon_d = \varepsilon_d^*$. Then we can easily see that the evaluated value of LHS is always larger than one. It means that the both sides of Eq. (22) can not be equal for any $\beta \in [0, 1]$. (QED)

This lemma suggests that one-sided decision always undermines the efficiency of investment. Whenever one party recoups the return from the investment made by the other party, it brings about such a hold-up problem.

4 Labour Policies

Following Sato and Sugiura (2003), we investigate the effect of the introduction of labour policies such as a subsidy to investment in firm-specific skills and unemployment benefit to our model.

Let $h\ (h<c)$ denote a subsidy to a unit investment in firm-specific skills and B be unemployment benefit. For analytical simplicity, the financial resources of labour policies are not considered.[8] Replacing c and b respectively by $c-h$ and $b+B$ in the equations from (13) to (15), modified version of skill accumulation, job creation, and job destruction conditions are respectively obtained as follows.

$$\frac{1-\beta}{r+sG(\hat{\varepsilon_d})} f_1(\hat{t}, z) = c-h, \tag{23}$$

$$\frac{1-\beta}{r+s}(\varepsilon_u - \hat{\varepsilon_d}) - (c-h)\hat{t} = \frac{k}{q(\hat{\theta})}, \tag{24}$$

$$f(\hat{t}, z) + \frac{r+sG(\hat{\varepsilon_d})}{r+s}\hat{\varepsilon_d} + \frac{s}{r+s}\int_{\hat{\varepsilon_d}}^{\varepsilon_u} x dG(x)$$
$$= b+B+q(\hat{\theta})\hat{\theta}\cdot\frac{\beta}{r+s}(\varepsilon_u - \hat{\varepsilon_d}). \tag{25}$$

where the market equilibrium with labour policies are denoted by a tuple $(\hat{t}, \hat{\varepsilon}_d, \hat{\theta})$. The determination of unemployment rate is unchanged. Using from Eqs. (23) to (25), we apply a comparative static analysis to examine the effects of the labour policies. Then we can obtain the following results.

Proposition 3 *An increase in a subsidy to a unit investment in firm-specific skills raises the amount of investment and reduces job destructions, if endogenous level of the investment is not so large. It, however, reduces the amount of investment and increases job destructions, if the level of the investment is large enough. In both cases, an increased subsidy gives an obscure effect on the market tightness*.

Proof See the Appendix F.

Since subsidy to investment in firm-specific skills encourages skill formation, it contributes to maintaining the current match in order to recoup the increased rent. It requires, however, a certain condition to obtain these results. If the endogenous level of investment is large enough, the amount of subsidy becomes so large that job destructions tend to occur. It is because a large subsidy lowers the opportunity cost of hoarding the current match by reducing the investment cost on the next hiring. Let us focus on unemployment benefit as a labour policy. The following result can be established.

Proposition 4 *An increase in unemployment benefit reduces the amount of investment and raises job destructions. Furthermore, it lowers the market tightness and raises the unemployment rate.*

Proof See the Appendix F.

Unemployment benefit gives an opposite effect on the equilibrium from the previous case, but the endogenous level of investment does not interrupt the effect. It increases the bargained wage by pushing up worker's threat point. Then job destructions tend to occur, since the rent from the match is decreased. Firms reduce the investment in firm-specific skills, as the relationship between firm and worker becomes more fragile. The benefit to the unemployed workers harms firm's job creation. As we will see, we utilise it to avoid the inefficiency generated by the match externality.

We examine the characteristics of the labour policy package is like that can lead to the social optimum. After some calculations, we obtain the following proposition.

Proposition 5 *Appropriate labour policy set* $(\hat{h}^*, \hat{B}^*)$ *can make the equilibrium coincide with the optimum*:

$$\hat{h}^* = \left[\frac{s+\delta}{(r+s)(sG(\hat{\varepsilon}_d^*)+\delta)} - \frac{1-\beta}{r+sG(\hat{\varepsilon}_d^*)}\right] f_1(\hat{t}^*, z) > 0, \quad (26)$$

$$\hat{B}^* = \left[-q(\hat{\theta}^*)\hat{\theta}^*\beta - q'(\hat{\theta}^*)\hat{\theta}^{*2}\right] \cdot \left(\frac{\varepsilon_u - \hat{\varepsilon}_d^*}{r+s}\right) + q'(\hat{\theta}^*)\hat{\theta}^{*2}c\hat{t}^* \quad (27)$$

Let

$$\hat{\beta}^* \equiv \eta(\hat{\theta}^*) \cdot \left[1 - \frac{(r+s)c\hat{t}^*}{\varepsilon_u - \hat{\varepsilon}_d^*}\right]. \quad (28)$$

If $\beta < \hat{\beta}^*$, *then* $\hat{B}^* > 0$ *and vice versa.*
If $\beta = \hat{\beta}^*$, *then* $\hat{B}^* = 0$.

Proof We can obtain the conditions shown above by comparing the equations from (19) to (21) with those from (23) to (25), where all endogenous variables are evaluated at the same values. They are denoted by a tuple $(\hat{t}^*, \hat{\varepsilon}_d^*, \hat{\theta}^*)$, which means the social optimum when the labour policies are available. The inequity of Eq. (26) can be easily obtained by comparing the right hand side with those evaluated at $\beta = 0$. (QED)

Hosios (1990) and Pissarides (1990) have shown that the equilibrium rate of job creation is inefficient when there are search frictions, that is there exists the match externality. They found that the inefficiency will not arise only when the elasticity of the matching function is equal to the bargaining power of workers. We have obtained a modified version of the Hosios-Pissarides condition as is shown in Eq. (28). The optimal bargaining power of workers must be reduced from that of the standard model, because firms incur the investment cost whenever a new match begins. This property is pecular to firm-specific investment, since it relis on the nature that the investment cost is always a burden to a new match. Subsidy to investment is always required, because one-sided investment decision causes underinvestment as Lemma 1 suggests. The tightness of the market do not directly affect the required level of subsidy to firm-specific investment. This property contrasts with Sato and Sugiura (2003)'s result, which showed that the market tightness give a direct effect on the required subsidy to general skills. Unemployment benefit is utilised to remedy the inefficiency generated by the match externality. Theoretically, the unemployed workers should be subsidised or taxed, according to the bargaining power of the workers.

5 Conclusion

We have analysed the characteristics of investment in firm-specific skills under search friction and explore how we can remedy the inefficiency generated by both one-sided investment decision and the match externality.

In general, subsidy to firm-specific skill acquisition bring about favorable effects by motivating skill formation and reducing job destructions. When the endogenous level of the investment is large, an increased subsidy may cause adverse effects. Since it lowers the opportunity cost of hoarding the current match, it contributes to increasing job destructions.

We also have derived a modified version of Hosios-Pissarides condition for optimal bargaining power of workers. The investment in firm-specific skills lowers the worker's optimal bargaining power. An appropriate combination of subsidy to skills and unemployment benefit can attain the optimum even when there exists investment in firm-specific skills. We believe that our analysis have complemented the issues on labour policies, especially on the equilibrium search and matching approach with skill acquisition.

Appendix

A Derivation of Eq. (7)

Combining Eq. (2) with Eq. (3) give

$$(r+s)J(t,z,\varepsilon) = f(t,z) + \varepsilon - w(t,z,\varepsilon) + s\left[\int_{\varepsilon_d}^{\varepsilon_u} J(t,z,x)dG(x)\right]. \quad (29)$$

Replacing the relationship $J(t,z,x) = \frac{1-\beta}{\beta} \times \{W(t,z,x) - U(z)\}$ derived from Eq. (6) into Eq. (29) gives the following equation.

$$(r+s)W(t,z,\varepsilon) - rU(z) = \frac{\beta}{1-\beta}[f(t,z)+\varepsilon - w(t,z,\varepsilon)] + s\left[\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) + G(\varepsilon_d)U(z)\right]. \quad (30)$$

Simplifying Eq. (4), we obtain

$$(r+s)W(t,z,\varepsilon) = w(t,z,\varepsilon) + s\left[\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) + G(\varepsilon_d)U(z)\right]. \quad (31)$$

Then, combining Eq. (30) with (31) yield

$$w(t,z,\varepsilon) = \beta[f(t,z)+\varepsilon] + (1-\beta)rU(z). \quad (32)$$

(QED)

B Derivation of Eq. (8)

Multiplying the both sides of Eq. (31) by $g(\varepsilon)$, taking integrals of both sides with respect to ε from $\varepsilon = \varepsilon_d$ to $\varepsilon = \varepsilon_u$ and modifying a notation give

$$(r+s)\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) = \int_{\varepsilon_d}^{\varepsilon_u} w(t,z,x)dG(x) + s\left[1-G(\varepsilon_d)\right]\left[\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) + G(\varepsilon_d)U(z)\right].$$

Then, we obtain

$$\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) = \frac{1}{r+sG(\varepsilon_d)} \times \left[\int_{\varepsilon_d}^{\varepsilon_u} w(t,z,x)dG(x) + s\left[1-G(\varepsilon_d)\right]G(\varepsilon_d)U(z)\right]. \quad (33)$$

We obtain the following equation from Eqs. (31) and (33).

$$\begin{aligned}(r+s)W(t,z,\varepsilon) &= w(t,z,\varepsilon) + s\int_{\varepsilon_d}^{\varepsilon_u} W(t,z,x)dG(x) + sG(\varepsilon_d)U(z)\\ &= \beta\left[f(t,z)+\varepsilon\right] + (1-\beta)rU(z) \qquad (34)\end{aligned}$$

$$+\frac{s}{r+sG(\varepsilon_d)}\left[\int_{\varepsilon_d}^{\varepsilon_u} w(t,x)dG(x) + s\left[1-G(\varepsilon_d)\right]G(\varepsilon_d)U(z)\right] + \frac{s}{r+sG(\varepsilon_d)}[r+sG(\varepsilon_d)]G(\varepsilon_d)U(z). \quad (35)$$

Note that we utilise the wage equation (32) at the second equal sign.

$$(r+s)W(t,z,\varepsilon) = \beta\left[f(t,z)+\varepsilon\right] + (1-\beta)rU(z) + \frac{s}{r+sG(\varepsilon_d)}\left[\int_{\varepsilon_d}^{\varepsilon_u} w(t,x)dG(x) + (r+s)G(\varepsilon_d)U(z)\right] \quad (36)$$

Here, using the following relationship that is derived by integrating both sides of Eq. (32),

$$\int_{\varepsilon_d}^{\varepsilon_u} w(t,z,x)dG(x) = (1-G(\varepsilon_d))\left[\beta f(t,z) + (1-\beta)rU(z)\right] + \beta\int_{\varepsilon_d}^{\varepsilon_u} xdG(x),$$

we obtain

$$(r+s)W(t,z,\varepsilon) = \frac{r+s}{r+sG(\varepsilon_d)}\left[\beta f(t,z) + \{(1-\beta)r + sG(\varepsilon_d)\}U(z)\right] + \frac{s}{r+sG(\varepsilon_d)}\beta\int_{\varepsilon_d}^{\varepsilon_u} xdG(x) + \beta\varepsilon. \quad (37)$$

From this equation, we have Eq. (8). (QED)

C Derivation of Eq. (12)

Evaluating both sides of Eq. (8) at $\varepsilon = \varepsilon_d$ gives

$$W(t,z,\varepsilon_d) = U(z) = \frac{1}{r+sG(\varepsilon_d)}\left[\beta f(t,z) + \{(1-\beta)r + sG(\varepsilon_d)\}U(z)\right] + \frac{1}{r+s}\left\{\frac{s}{r+sG(\varepsilon_d)}\beta\int_{\varepsilon_d}^{\varepsilon_u} xdG(x) + \beta\varepsilon_d\right\}. \quad (38)$$

Transforming the equation above gives

$$\frac{\beta r}{r+sG(\varepsilon_d)}U(z) = \frac{\beta f(t,z)}{r+sG(\varepsilon_d)} + \frac{\beta\varepsilon_d}{r+s} + \frac{s\beta}{\{r+sG(\varepsilon_d)\}(r+s)}\int_{\varepsilon_d}^{\varepsilon_u} xdG(x). \quad (39)$$

We find the following equation from Eqs. (2) and (6).

$$J(t,z,\varepsilon) = \frac{1-\beta}{\beta}\{W(t,z,\varepsilon) - U(z)\}. \quad (40)$$

We substitute Eq. (8) into this equation gives

$$J(t,z,\varepsilon) = \frac{1-\beta}{r+sG(\varepsilon_d)}f(t,z) + \frac{(1-\beta)\varepsilon}{r+s} + \frac{1-\beta}{r+sG(\varepsilon_d)}\left\{\frac{s}{r+s}\int_{\varepsilon_d}^{\varepsilon_u} xdG(x) - rU(z)\right\}. \quad (41)$$

Evaluating the both sides of Eq. (41) at $\varepsilon = \varepsilon_d$ gives Eq. (12). (QED)

D Proof of Proposition 1

Let $|D|$ denote the determinant of the coefficient matrix at the right hand side of Eq. (17). Then we obtain

$$|D| = A_1 \cdot \begin{vmatrix} \frac{-(1-\beta)}{r+s} & \frac{kq'}{q^2} \\ B_2 & B_3 \end{vmatrix} + A_2\left\{\frac{\beta c}{r+s}(\varepsilon_u - \varepsilon_d)\left(q'(\theta)\theta + q(\theta)\right) - f_1\frac{kq'}{q^2}\right\}.$$

Here, the determinant of the first term is positive and the large bracket of the second term is also positive. Therefore, $|D| < 0$ holds because $A_1 < 0$ and $A_2 < 0$. Then we have the following equations from Cramer's formula.

$$\frac{dt}{dz}=\frac{1}{|D|}\begin{vmatrix}\frac{-(1-\beta)}{r+sG(\varepsilon_d)}f_{12} & A_2 & 0\\ 0 & \frac{-(1-\beta)}{r+s} & \frac{kq'}{q^2}\\ -f_2 & B_2 & B_3\end{vmatrix}=\frac{1}{(-)}\begin{vmatrix}- & - & 0\\ 0 & - & -\\ - & + & -\end{vmatrix}>0, \tag{42}$$

$$\frac{d\varepsilon_d}{dz}=\frac{1}{|D|}\begin{vmatrix}A_1 & \frac{-(1-\beta)}{r+sG(\varepsilon_d)}f_{12} & 0\\ -c & 0 & \frac{kq'}{q^2}\\ f_1 & -f_2 & B_3\end{vmatrix}=\frac{1}{(-)}\begin{vmatrix}- & - & 0\\ - & 0 & -\\ + & - & -\end{vmatrix}<0, \tag{43}$$

$$\frac{d\theta}{dz}=\frac{1}{|D|}\begin{vmatrix}A_1 & A_2 & \frac{-(1-\beta)}{r+sG(\varepsilon_d)}f_{12}\\ -c & \frac{-(1-\beta)}{r+s} & 0\\ f_1 & B_2 & -f_2\end{vmatrix}. \tag{44}$$

We can obtain the last inequity of Eq. (42) from the signs of all the elements, even if there is no complementarity between general and firm-specific skills, that is $f_{12}=0$. Especially, if it exists, that is $f_{12}>0$, it strengthen thisinequity. The last sign of Eq. (43) is obtained in the same way. To determine the sign of Eq. (44), we transform this equation using Eqs. (13) and (16). Then we have

$$\begin{aligned}\frac{d\theta}{dz} &= \frac{1}{|D|}\left\{-f_2\begin{vmatrix}A_1 & A_2\\ -c & \frac{-(1-\beta)}{r+s}\end{vmatrix}-\frac{(1-\beta)}{r+sG(\varepsilon_d)}f_{12}\begin{vmatrix}-c & \frac{-(1-\beta)}{r+s}\\ f_1 & B_2\end{vmatrix}\right\}\\ &= \frac{1}{|D|}\left\{A_1f_2\cdot\frac{1-\beta}{r+s}-cf_2A_2+cf_{12}\cdot\frac{\beta q(\theta)\theta}{r+sG(\varepsilon_d)}\cdot\frac{1-\beta}{r+s}\right\}\\ &= \frac{1-\beta}{|D|(r+s)(r+sG(\cdot))}\Big\{(1-\beta)f_{11}f_2\\ &\quad +(1-\beta)^{-1}s(r+s)g(\cdot)c^2f_2+f_{12}\beta q(\theta)\theta c\Big\}.\end{aligned}$$

Noting that $|D|<0$, let us examine this equation. The first term in the bracket is negative, while the second and the third terms are positive. If determined amount of investment is not so large, the absolute value of f_{11} is large. Then we can expect that the first term dominate against the second and the third terms. It means that the sign of $\frac{d\theta}{dz}$ will be positive. If determined amount of investment is large enough, the absolute value of f_{11} is small and the sign of $\frac{d\theta}{dz}$ will be negative. (QED)

E Derivation of Social Optimality Conditions

We define the present value Hamiltonian as follows.

$$\begin{aligned}\mathcal{H} &= e^{-\rho\tau}(y+bu-k\theta u-q(\theta)\theta u\cdot ct)\\ &+ \mu_1\left[sG(\varepsilon_d)\cdot(1-u)-\{\theta q(\theta)+\delta\}u\right]+\mu_2\theta q(\theta)u\{f(t,z)+\varepsilon_u\}\\ &+ \mu_2\left[s(1-u)\int_{\varepsilon_d}^{\varepsilon_u}(f(t,z)+\varepsilon)g(\varepsilon)d\varepsilon-(s+\delta)y\right],\end{aligned} \tag{45}$$

where μ_1 and μ_2 denote the costate variables that accompany u and y respectively. The state variables obey the following equations.

$$\begin{aligned}\frac{\partial\mathcal{H}}{\partial u}=-\dot{\mu_1} \quad\Leftrightarrow\quad & e^{-\rho\tau}(b-k\theta-q(\theta)\theta\cdot ct)+\mu_1\cdot[-sG(\varepsilon_d)-\theta q(\theta)-\delta]\\ &+ \mu_2\cdot\left[\theta q(\theta)\{f(t,z)+\varepsilon_u\}-s\int_{\varepsilon_d}^{\varepsilon_u}(f(t,z)+\varepsilon)g(\varepsilon)d\varepsilon\right]+\dot{\mu_1}=0,\end{aligned} \tag{46}$$

$$\frac{\partial\mathcal{H}}{\partial y}=-\dot{\mu_2} \quad\Leftrightarrow\quad e^{-\rho\tau}-(s+\delta)\mu_2+\dot{\mu_2}=0. \tag{47}$$

The control variables need to satisfy the following conditions.

$$\begin{aligned}\frac{\partial\mathcal{H}}{\partial\theta}=0 \quad\Leftrightarrow\quad & e^{-\rho\tau}\left[-ku-\{q'(\theta)\theta u+q(\theta)u\}ct\right]-\mu_1\{q'(\theta)\theta u+q(\theta)u\}\\ &+\mu_2\{q'(\theta)\theta u+q(\theta)u\}\{f(t,z)+\varepsilon_u\}=0.\end{aligned} \tag{48}$$

$$\frac{\partial\mathcal{H}}{\partial\varepsilon_d}=0 \quad\Leftrightarrow\quad \mu_1sg(\varepsilon_d)\cdot(1-u)-\mu_2s(1-u)[f(t,z)+\varepsilon_d]g(\varepsilon_d)=0. \tag{49}$$

$$\begin{aligned}\frac{\partial\mathcal{H}}{\partial t}=0 \quad\Leftrightarrow\quad & e^{-\rho\tau}\{-q(\theta)\theta uc\}\\ &+\mu_2[\theta q(\theta)uf_1+s(1-u)\{1-G(\varepsilon_d)\}f_1]=0.\end{aligned} \tag{50}$$

Let us organise the necessary conditions that are derived above. From Eq. (47), we obtain

$$\mu_2=\frac{e^{-\rho\tau}}{\rho+\delta+s}=\frac{e^{-\rho\tau}}{r+s}. \tag{51}$$

Eq. (49) gives

$$\mu_1=[f(t^*,z^*)+\varepsilon_d^*]\cdot\mu_2. \tag{52}$$

From Eqs. (16), (50) and (51), we obtain Eq. (19). Combining Eq. (48) with Eqs. (51) and (52) gives Eq. (20). Next, combining Eq. (46) with Eqs. (51) and (52) generates Eq. (21). The equilibrium rate of unemployment is obtained from the definition of unemployment flow. (QED)

F Proof of Propositions 3 and 4

Totally differentiating the equations from (23) to (25) gives

$$\begin{bmatrix} A_1 & A_2 & 0 \\ -(c-h) & \frac{-(1-\beta)}{r+s} & \frac{kq'}{q^2} \\ f_1 & B_2 & B_3 \end{bmatrix} \begin{bmatrix} d\hat{t} \\ d\hat{\varepsilon}_d \\ d\hat{\theta} \end{bmatrix} = \begin{bmatrix} -1 \\ -\hat{t} \\ 0 \end{bmatrix} dh + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} dB, \quad (53)$$

where the notations are defined at Section 2 and these are evaluated at a tuple $(\hat{t}, \hat{\varepsilon}_d, \hat{\theta})$. Let $|D|$ denote the determinant of the coefficient matrix at the right hand side of Eq. (53). We can obtain $|D| < 0$ in the same way as is shown in the Appendix D. Then Cramer's formula gives the following equations.

$$\frac{d\hat{t}}{dh} = \frac{1}{|D|}\left\{\frac{1-\beta}{r+s}B_3 + \frac{kq'}{q^2}B_2 + \hat{t}A_2B_3\right\}, \quad (54)$$

$$\frac{d\hat{\varepsilon}_d}{dh} = \frac{1}{|D|}\left\{-(c-h)B_3 - \frac{kq'}{q^2}f_1 - \hat{t}A_1B_3\right\}, \quad (55)$$

$$\frac{d\hat{\theta}}{dh} = \frac{1}{|D|}\left\{(c-h)B_2 - \frac{1-\beta}{r+s}f_1 + \hat{t}A_1B_2 - \hat{t}A_2f_1\right\}. \quad (56)$$

If endogenous level of $\hat{t}$ is not so large, we obtain $\frac{d\hat{t}}{dh} > 0$ and $\frac{d\hat{\varepsilon}_d}{dh} < 0$, because the first and second terms dominates the third term in the bracket in (54) and (55) respectively. If the level of $\hat{t}$ is large enough, we obtain the opposite results such as $\frac{d\hat{t}}{dh} > 0$ and $\frac{d\hat{\varepsilon}_d}{dh} < 0$, because of the opposite reasons to those given above. We also have the following equations in the same way as before.

$$\frac{d\hat{t}}{dB} = \frac{1}{|D|}A_2\frac{kq'}{q^2} < 0, \quad (57)$$

$$\frac{d\hat{\varepsilon}_d}{dB} = \frac{-1}{|D|}A_1\frac{kq'}{q^2} > 0, \quad (58)$$

$$\frac{d\hat{\theta}}{dB} = \frac{1-\beta}{|D|(r+sG(\cdot))}\left\{-\frac{(1-\beta)f_{11}}{r+s} + sg(\cdot)f_1c\right\} < 0. \quad (59)$$

(QED)

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Notes

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1) Our model is motivated from the works of Mortensen and Pissarides (1994) and Shintoyo (1999). For reference to standard equilibrium search model, see Mortensen and Pissarides (1999b,c) and Pissarides (2000).

2) Mortensen and Pissarides (1999a) conduct a rigorous analysis of the labour market that is segmented by the skill without its observability.

3) We do not focus on the problem which party should pay for training cost. Shintoyo (1999) assumed that a worker pays for firm-specific skills, contrary to our model. The analysis of the training cost problem is given by Loewenstein and Spletzer (1998).

4) This assumption is line with that of Mortensen and pissarides (1994).

5) The firm makes optimal investment by taking the value of $U(z)$ as given, because skill specificity makes the value independent from the firm's investment. At the equilibrium, the value depends on the investment level chosen by all the firms, $\bar{i}$. Similar argument can be found in Pissarides (2001). The second-order condition for this problem is always satisfied from the assumptions.

6) It means that $sG(\varepsilon_d)\cdot(1-u)+\delta=[\theta q(\theta)+\delta]u$.

7) For notational simplicity, we omit the time indices.

8) When we introduce a lump-sum tax into our model, we can obtain the same results as those are derived in this paper.

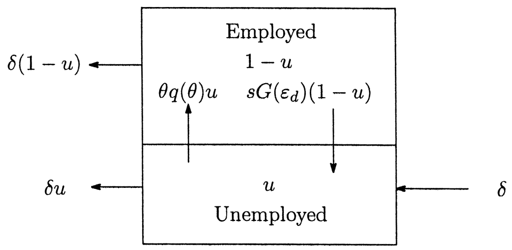

Figure 1: Flow of workers