Individuals' Attitudes toward Pay-as-You-Go Pension Systems and Longevity in an Overlapping Generations Model with Endogenous Fertility

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Abstract

In the context of population aging in OECD countries, this study explores the effects of changes in longevity and pension contribution rate on fertility rate and capital accumulation using a three-period overlapping generations model with pay-as-you-go (PAYG) and a defined contribution pension scheme. One feature of our study is that individuals maximize their utility by considering that the number of children affects the amount of their pension benefits. The study diverges from the previous studies in that in previous studies, individuals maximized utility considering their pension benefits as given. In recent years, it has been shown that individuals have a strong understanding of the relationship between pension systems and population size. Against this backdrop, under sufficiently high pension contributions, we find that longevity leads to increased fertility. This is because the effect of

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longevity of decreasing individual pension benefits is offset by individuals' having more children. This result contrasts with previous studies showing that longevity decreases fertility. Our results suggest that governments facing inevitable longevity can mitigate the decline in fertility rates by strengthening public relations activities in the pension system.

Keywords: PAYG Pensions, Endogenous Fertility, Capital Accumulation, Aging, Longevity

(JEL: D15, E21, E23, H55, J13, J18, O41)

1. Introduction

Many advanced countries are experiencing aging populations. The causes for this are a rising life expectancy and declining fertility rates. In OECD countries, life expectancy has continued to rise to date. For example, as shown in Figure 1, life expectancy has been rising since 1975 in Japan, Germany, and Italy and is expected to continue to rise. In addition, several countries have experienced declines in their total fertility rates, with the total fertility rate falling well below 2.08, the level required to maintain a population level. Figure 2 shows that the total fertility rates in major advanced countries are projected to remain below 2.00.

According to OECD (2019), many advanced countries with aging populations have adopted a pay-as-you-go (PAYG) pension system. Under this system, pension contributions are collected from the working population, and pension benefits are paid to the elderly. In addition, many countries have adopted a defined-benefit (DB) pension system in which pension benefits are determined based on the number of years of contribution and income. Under the DB PAYG pension system, the rapid economic and population growth in OECD countries in the second half of the 20th century led to a dramatic increase in the rate of return of pension systems. However, the system is vulnerable to demographic changes, and the future pension benefits strongly depend on future demographic changes. According to OECD (2023), to cope with population aging, PAYG DC pension systems have been introduced in OECD countries in Italy, Latvia, Norway, Poland, and Sweden, where pension benefit levels are based on the contributions paid.

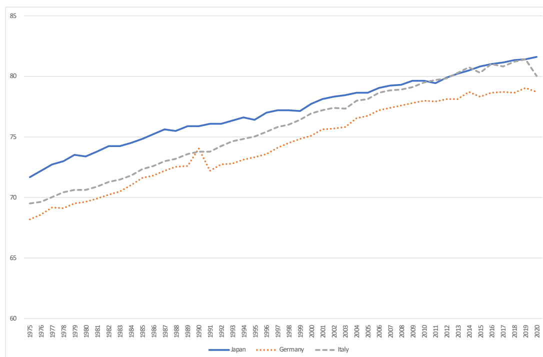

Figure 1 Life expectancy

Source: Prepared by the author based on OECD statistics.

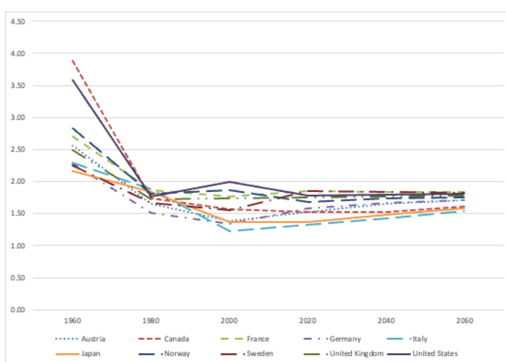

Figure 2 Total fertility rate

Source: Prepared by the author based on OECD (2023).

In addition to pension reforms, individuals' attitudes toward pension systems have changed. Surveys in Japan have shown that the understanding of the pension system has deepened in recent years. The CabinetOffice (2018b) conducted a "Public Opinion Survey on Retirement Planning and Public Pensions." According to this survey, the number of respondents who recognized that the system was designed to allow the working-age population to support the elderly receiving pensions has increased with each survey. In the 2018 survey, 67.1% of the respondents recognized this. In Europe, many individuals are alarmed about demographic changes. For example, according to the EU (2023), 67% of its citizens agree that demographic changes undermine the long-term sustainability of public finance. In other words, individuals are likely to recognize that future pension benefits depend on the population size.

Against this backdrop, we study how demographics and pension systems interact. We examine the effects of longevity on capital accumulation and fertility under the assumption that individuals have a high level of understanding of the relationship between pension benefits and population size. We employ the model proposed by Cipriani and Fioroni (2019). This is a three-period overlapping generations model that incorporates endogenous fertility and a DC PAYG pension system. It is a highly tractable model for studying capital accumulation and demographics, ignoring the bequest motive with respect to individuals' fertility choices.[1] The unique feature of our study is that individuals make decisions based

[1] Cipriani and Fioroni (2019) present a model with and without elderly labor, and this study employs models without elderly labor. In recent years, several studies have incorporated phenomena closely related to demographic changes, such as elderly labor, human capital accumulation, child care, and elder care, into models dealing with pension policy. This study ignores these extensions made in these studies to focus on the interaction between individuals' fertility and their attitudes toward the pension systems. See Fanti (2015), Tanaka (2018), Dedry, Onder and Pestieau (2017), Cipriani (2018), Cipriani and Pascucci (2018), Miyake and Yasuoka (2018), Liu and Thøgersen (2019), Hirono and Mino (2021), and Cipriani and Fioroni (2023) for studies on pension policy incorporating elderly labor. See Cremer, Gahvari and Pestieau (2011), Cipriani and Makris (2012), and Cipriani and Fioroni (2023) for studies of pension policy incorporating human capital accumulation. See van Groezen, Leers and Meijdam (2003), Hirazawa and Yakita (2009), and Yasuoka and Miyake (2014) for studies on pension policy incorporating childcare policy. See Yasuoka (2020) for studies on pension policy incorporating elderly care.

on the fact that the amount of pension benefits they receive in old age depends on their number of children. This is appropriate because the number of children an individual has is the same as the number of children in a generation under the assumption of homogeneous individuals. [2] Cipriani and Fioroni (2019) also assume homogeneous individuals, but they assume individuals to make decisions with future pension benefits as given. [3] As noted above, given that individuals have a strong understanding of the relationship between pension systems and population size, it is necessary to examine the effects of longevity on capital accumulation and fertility if individuals make decisions considering that their future pension benefits depend on the number of their children. [4]

In this study, we employ the model proposed by Cipriani and Fioroni (2019). This model is superior to other models in examining the relationship between demographics and modern pension systems for three reasons. First, the formulation of fertility choices is simple. In this model, the only motivation for an individual's fertility is the utility derived from the number of children. By contrast, the models of Zhang (1995), Zhang and Zhang (1998), and Wigger (1999) incorporate a bequest motive behind individual fertility choices. Parents gift legacies with the expectation of support from their children. Additionally, in Barro and Becker (1989)'s model, parental utility depends on the children's utility. Our study ignores the

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[2] This study assumes homogeneous individuals and does not result in a freeride to increase pension benefits due to the children of others discussed in Cremer, Gahvari and Pestieau (2008).

[3] See Wigger (1999), Cipriani (2014), Cipriani (2018), Cipriani and Pascucci (2018), and Cipriani and Fioroni (2019) for studies with a similar framework where individuals maximize utility function considering pension benefits as given. In models such as these with a fertility rate of endogenous, results change when individuals maximize utility function subject to their pension benefits. In the models with a fertility rate of exogenous such as Fanti and Gori (2012) and Fanti (2015), the result does not change even if individuals maximize utility function subject to their pension benefits.

[4] We assume homogeneous individuals and do not discuss "free-riding problems" such as others' increasing pension benefits by having more children. See Cremer, Gahvari and Pestieau (2008) for a discussion of the "free-riding problem" in pension policy.

bequest motive, and to focus on the demographic effects of longevity and changes in pension benefits, employs a simple model in which parental utility depends only on the number of children.

Second, this model is a set of PAYG pension systems. Zhang (1995) and Kemnitz and Wigger (2000) compared PAYG and funded pension systems. Most modern advanced countries maintain PAYG pension systems. Therefore, employing a model adapted to modern pension systems is necessary.

Third, the model incorporates endogenous fertility and survival uncertainty into old age using DC pension systems. Due to the risks associated with maintaining PAYG pension systems, some countries have adopted DC PAYG pension systems. Fanti and Gori (2012) developed one of the simplest models using the DC PAYG pension system. They concluded that a decline in fertility does not necessarily reduce pension benefits because it promotes capital accumulation per capita. However, they set fertility as exogenous, and did not explicitly incorporate a measure of longevity into their model. The Cipriani and Fioroni (2019) model employed in this study extends the Fanti and Gori (2012) model by incorporating endogenous fertility and probability of survival into old age, which represents the degree of longevity, while using the DC PAYG pension system. [5] These extensions enable us to show that aging reduces pension benefits and capital accumulation.

The results of this study are as follows: First, we find lower per worker capital accumulation in the steady state than in Cipriani and Fioroni (2019), and a higher fertility

[5] See also Yakita (2001) and Cipriani (2014) for studies incorporating endogenous fertility and the probability of survival into old age.

rate in the steady state than in Cipriani and Fioroni (2019). This is because of increasing pension benefits with respect to fertility choices in this study. In other words, the incentives for fertility are stronger than those in Cipriani and Fioroni (2019). This discourages savings and increases the population in this study more than in Cipriani and Fioroni (2019), in turn, lowering capital accumulation per worker at the steady state. Second, when the pension contributions are sufficiently high, longevity mitigates the effect of declining the steady-state fertility rate. In this study, individuals can decide to increase fertility to mitigate future declines in pension benefits due to longevity. The higher the pension contribution rate, the greater the effect of the number of children on increasing pension benefits. Consequently, when the pension contributions are sufficiently high, longevity mitigates the effect of declining the fertility rate. In Cipriani and Fioroni (2019), individuals made decisions by taking pension benefits as given; therefore, they did not attempt to mitigate the decrease in pension benefits. Therefore, Cipriani and Fioroni (2019) have shown that longevity increases consumption preferences in old age, which can decrease fertility. Our results suggest that enhancing individuals' pension knowledge and awareness can mitigate the decline in fertility rates. One means of achieving this is, for example, to strengthen pension education in schools and other institutions, as well as advertising of pension system. In addition, the government must set tax rates high enough to have the effect of increasing the fertility rate.

The remainder of this paper is organized as follows. The model is described in the following section. In Section 3, market equilibrium is examined and steady states are derived for capital accumulation, fertility, and savings. Section 4 presents the comparative statics in the steady state. The effects of increasing longevity on capital accumulation, fertility, and savings as well as the effects of increasing taxes on capital accumulation and savings are examined. In Section 5, numerical simulations are performed for analyses that yielded ambiguous results in Section 4. Section 6 concludes the paper.

2. Model

We present a three-period overlapping generations model with endogenous fertility and a pay-as-you-go (PAYG) pension system developed by Cipriani and Fioroni (2019). In this model, individuals live potentially for three periods. In the first period, they are young and make no decisions. In the second period, they are adults and earn a labor income w_t. They choose consumption and the number of children n_t. In the third period, they are old. They survive from adulthood into old age with probability π. They retire, receive pension benefits p_{t+1}, and enjoy consumption. We name individuals who are adults in period t as generation t. We denote the population size of generation t by N_t. As adult individuals of generation $t-1$ grow old in period t, the population size of the elderly in period t is represented by N_{t-1}. Then, we define the ratio of the sizes of the adjacent generations as $\bar{n}_{t-1} = N_t/N_{t-1}$. [6] It is noteworthy that $\bar{n}_{t-1}$ is equal to n_{t-1} because the individuals are homogenous; $\bar{n}_{t-1}$ can be labeled the average fertility of generation $t-1$, $n_{t-1} N_{t-1}/N_{t-1}$. Next, we describe the behaviors of the government, individuals, and firms.

[6] If individuals are not homogeneous, the number of children an individual chooses to have does not correspond to the fertility rate of the economy. See Brunner (1996) and Cremer, Gahvari and Pestieau (2008) for a study on pension policy with different individuals.

2.1. Government

The government collects contributions $\tau \in (0,1)$ to finance public pensions under PAYG and defined contribution (DC) schemes. In the DC scheme, the government collects proportional contributions from adults in every period and distributes pension benefits p_t to the elderly in period t. It determines the pension benefits to satisfy the following budget constraints:

$$w_t \tau N_t = p_t \pi N_{t-1}. \tag{1}$$

By rewriting (1), we obtain,

$$p_t = \frac{w_t \tau \bar{n}_{t-1}}{\pi}. \tag{2}$$

Pension benefits depend on the population of each generation and the probability of survival from adulthood into old age.

2.2. Individuals

Individuals live in three periods: children, adults, and the elderly. The utility function of the individuals in generation t is given by:

$$U_t = \ln c_{a,t} + \gamma \ln n_t + \beta\pi \ln c_{o,t+1}, \tag{3}$$

where $c_{a,t}$ denotes consumption in adulthood, n_t, the number of children, and $c_{o,t+1}$, consumption in old age. The parameter γ measures the relative preference concerning the number of children; $\beta \in (0,1)$ denotes the time-preference discount factor. The adulthood budget constraint for generation t is given by:

$$c_{a,t} = (1-\tau)w_t - qn_tw_t - s_t, \tag{4}$$

where $\tau \in (0,1)$ is the contribution rate determined by the social security system. Thus, $(1-\tau)w_t$ represents the adulthood disposable income. $q \in (0,1)$ denotes the cost of raising one child relative to parental wages. [7] s_t represents savings.

Because they retire in old age, the budget constraint in old age for generation t is:

$$c_{o,t+1} = \frac{R_{t+1}}{\pi} s_t + p_{t+1}, \tag{5}$$

where $\frac{R_{t+1}}{\pi}$ denotes the rate of return on savings. According to Cipriani (2014), when financial market intermediaries lend to or borrow from individuals, the corresponding rate must incorporate the risk involved due to the agents' uncertain lifetimes. Assuming, then, that the intermediaries operate under the conditions of perfect competition and that entry is

[7] Similar to this study, the formulation that the costs of child rearing rise proportionally to wages is also provided by Wigger (1999), Yakita (2001), Fanti and Gori (2012), Cipriani (2014), Cipriani (2018), Cipriani and Pascucci (2018), Cipriani and Fioroni (2019), Cipriani and Fioroni (2021), and Cipriani and Fioroni (2023). This can be interpreted in different ways. First, a child is raised in a nursery school. In this case, the cost of the school is proportional to wages, for example, child care fees in Japan are determined by income as described by CabinetOffice (2018a). Second, the higher the income, the more the spending on children. Third, the higher the income, the higher the opportunity cost of time spent on child rearing.

costless, we have this rate of return equal to $\frac{R_{t+1}}{\pi}$, where R_{t+1} is the risk-free rate of interest. p_{t+1} denotes the expected pension benefits.

Individuals choose n_t, s_t, $c_{a,t}$ and $c_{o,t+1}$ to maximize utility function (3) subject to (4), (5), and (2) taking the wage, interest, and contribution rates as given. [8] Solving the optimization problem for n_t, s_t, $c_{a,t}$ and $c_{o,t+1}$, then, yields:

$$n_t = \frac{\gamma R_{t+1}(1-\tau)w_t}{(qw_t R_{t+1} - \tau w_{t+1})(1+\gamma+\beta\pi)}, \tag{6}$$

$$s_t = \frac{(1-\tau)w_t[qw_t R_{t+1}\beta\pi - \tau w_{t+1}(\gamma+\beta\pi)]}{(qw_t R_{t+1} - \tau w_{t+1})(1+\gamma+\beta\pi)}, \tag{7}$$

$$c_{a,t} = \frac{(1-\tau)w_t}{1+\gamma+\beta\pi}, \tag{8}$$

$$c_{o,t+1} = \frac{R_{t+1}(1-\tau)w_t}{1+\gamma+\beta\pi}\beta. \tag{9}$$

In our study, individuals make decisions taking into account that pension benefits depend on the number of their own children. On the other hand, in Cipriani and Fioroni (2019), individuals make decisions by taking pension benefits as given. Let n_t^p, s_t^p, $c_{a,t}^p$ and $c_{o,t+1}^p$ be the solving the optimization problem for n_t, s_t, $c_{a,t}$ and $c_{o,t+1}$ when individuals make decisions with pension benefits as a given respectively:

[8] Under this formulation, the savings of individuals who did not survive from adulthood into old age are distributed to the remaining surviving individuals. This can also be interpreted as inheriting a bequest from a spouse. See also Cipriani (2018), Cipriani and Pascucci (2018), and Cipriani and Fioroni (2019) for the same formulation.

$$n_t^p = \frac{\gamma R_{t+1}(1-\tau)w_t}{qw_t R_{t+1}(1+\gamma+\beta\pi) - \tau\gamma w_{t+1}},$$

$$c_{a,t}^p = \frac{(1-\tau)w_t + \pi p_{t+1}}{1+\gamma+\beta\pi} = \frac{(1-\tau)w_t[qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1} + \tau\gamma w_{t+1} R_{t+1}]}{[qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}](1+\gamma+\beta\pi)},$$

$$s_t^p = \frac{(1-\tau)w_t[qw_t R_{t+1}\beta\pi - \gamma\tau w_{t+1}]}{qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}},$$

$$c_{o,t+1}^p = \frac{(1-\tau)w_t R_{t+1} + \pi p_{t+1}}{1+\gamma+\beta\pi}\beta$$

$$= \frac{(1-\tau)w_t R_{t+1}[qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1} + \tau\gamma w_{t+1}]}{[qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}](1+\gamma+\beta\pi)}\beta.$$

For the fertility rate, from (6), $n_t > n_t^p$ holds. The fertility rate, n_t is higher than in case of individuals make decisions with pension benefits as a given because individuals have a higher incentive to have children if they make decisions considering that pension benefits depend on the number of their own children than if they make decisions with pension benefits as a given.

From $\frac{c_{a,t}^p}{c_{a,t}} = \frac{qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1} + \tau\gamma w_{t+1} R_{t+1}}{qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}} > 1$, $\frac{c_{o,t+1}^p}{c_{o,t+1}} = \frac{qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1} + \tau\gamma w_{t+1}}{qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}} > 1$, and $s_t^p - s_t = \frac{\tau(1-\tau)w_t w_{t+1}\gamma(\tau w_{t+1} + qw_t R_{t+1})}{[qw_t R_{t+1}(1+\gamma+\beta\pi) - \gamma\tau w_{t+1}]} > 0$, under the condition that the individual makes decisions considering that pension benefits depend on the number of their own children $c_{a,t}$, s_t, and $c_{o,t+1}$ are lower than under the condition that the individual makes decisions with pension benefits as a given for these optimal solutions.

2.3 Firms

To maximize profits, firms produce homogeneous goods that can be used for both consumption and investment. The output is defined by the Cobb-Douglas production function:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}, \tag{10}$$

where K_t denotes physical capital, and $L_t = N_t$ represents labor in period t. $\alpha \in (0,1)$ and A represent production technology parameters. We define $k_t \equiv K_t/L_t$; output per worker is denoted by $y_t = Ak_t^{\alpha}$. The wage and gross interest rates are:

$$w_t = A(1-\alpha)k_t^{\alpha}, \tag{11}$$

$$R_t = \alpha A k_t^{\alpha-1}, \tag{12}$$

where capital depreciates completely in one period.

3. Market Equilibrium

The equilibrium condition for the goods market is:

$$K_{t+1} = N_t s_t. \tag{13}$$

Dividing K_{t+1} in (13) by $L_{t+1} = N_{t+1}$ and using $N_{t+1} = \bar{n}_t N_t$ with $\bar{n}_t = n_t$ we obtain capital per worker as follows:

$$k_{t+1} = \frac{s_t}{n_t}. \tag{14}$$

In the following equation, individuals are assumed to not be laborers in old age. The capital accumulation equation per worker is:

$$k_{t+1} = \frac{Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha + \tau(1-\alpha)] + \beta\pi\tau(1-\alpha)} k_t^{\alpha}. \tag{15}$$

From (15), $dk_{t+1}/dk_t > 0, d^2k_{t+1}/dk_t^2 < 0$.[9] Thus, there is one globally stable steady-state level of k as follows:

$$k^* = \left\{\frac{Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha + \tau(1-\alpha)] + \beta\pi\tau(1-\alpha)}\right\}^{\frac{1}{1-\alpha}}. \tag{16}$$

Proposition 1: Suppose individuals maximize lifetime utility considering that their pension benefits depend on the number of their own children. In this case, the steady-state level of capital per worker is lower than if individuals maximize lifetime utility with pension benefits

[9] See Appendix A for the derivation of (13).

as a given.

The steady-state level of capital accumulation per worker k^* is lower than that in Cipriani and Fioroni (2019) in which individuals maximize lifetime utility with pension benefits as a given. [10] As discussed later, this is because individuals have more children, reducing their savings. This finding reinforces the effect of the PAYG pension system in crowding out savings. Thus, maximizing utility by considering that an individual's pension benefit depends on the number of his or her own children negatively affects economic growth.

Next, we provide the steady-state level of consumption in adulthood and old age, fertility, and savings. [11] They are, in order, as follows:

$$c_a^* = \frac{(1-\tau)A(1-\alpha)k^{*\alpha}}{1+\gamma+\beta\pi}$$

$$= \frac{1-\tau}{1+\gamma+\beta\pi}[A(1-\alpha)]^{\frac{1}{1-\alpha}}\left[\frac{q\alpha\beta\pi}{\gamma\{\alpha+\tau(1-\alpha)\}+\beta\pi\tau(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}},$$

$$c_o^* = \frac{(1-\tau)A^2\alpha(1-\alpha)\beta k^{*2\alpha-1}}{1+\gamma+\beta\pi}$$

$$= \frac{1-\tau}{1+\gamma+\beta\pi}A^{\frac{1}{1-\alpha}}[(1-\alpha)\alpha\beta]^{\frac{\alpha}{1-\alpha}}\left[\frac{q\pi}{\gamma\{\alpha+\tau(1-\alpha)\}+\beta\pi\tau(1-\alpha)}\right]^{\frac{2\alpha-1}{1-\alpha}},$$

$$n^* = \frac{(1-\tau)\{\gamma[\alpha+\tau(1-\alpha)]+\beta\pi\tau(1-\alpha)\}}{q(1+\gamma+\beta\pi)[\alpha+\tau(1-\alpha)]}$$

$$= \frac{(1-\tau)\{\gamma[\alpha+\tau(1-\alpha)]+\beta\pi\tau(1-\alpha)\}}{q\alpha\beta\pi+q(1+\gamma)[\alpha+\tau(1-\alpha)]+q\beta\pi\tau(1-\alpha)}, \qquad (17)$$

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[10] In the Cipriani and Fioroni (2019), the steady-state level of capital per worker was $k_p^* = \left\{\frac{Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha+\tau(1-\alpha)]}\right\}^{\frac{1}{1-\alpha}}$.

[11] See Appendix B for the derivation of (17) and (18).

$$s^* = \frac{\alpha A\beta\pi(1-\tau)(1-\alpha)k^{*\alpha}}{\alpha\beta\pi + (1+\gamma)[\alpha + \tau(1-\alpha)] + \beta\pi\tau(1-\alpha)}. \quad (18)$$

Proposition 2: Suppose individuals maximize lifetime utility considering that their pension benefits depend on the number of their own children. The steady-state fertility rate then is higher than if individuals maximize lifetime utility with pension benefits as a given.

The steady-state fertility rate n^* is higher than that in Cipriani and Fioroni (2019) because individuals gain higher pension benefits by having more children. Compared to the Cipriani and Fioroni (2019), k^* is lower, and therefore the lifetime income of individuals is lower. Furthermore, n^* is higher than in the Cipriani and Fioroni (2019). As a result, s^* is lower than in Cipriani and Fioroni (2019). [12]

Finally, the steady-state level of pension benefits per elderly is:

$$p^* = \frac{w^*\tau n^*}{\pi} = \frac{A(1-\alpha)(k^*)^{\alpha-1}\tau n^*}{\pi}$$
$$= \tau A(1-\alpha)\left\{\frac{Aq\alpha\beta(1-\alpha)}{\gamma[\alpha + \tau(1-\alpha)] + \beta\pi\tau(1-\alpha)}\right\}^{\frac{\alpha}{1-\alpha}} \frac{n^*}{\pi^{(1-2\alpha)/(1-\alpha)}}. \quad (19)$$

[12] In Cipriani and Fioroni (2019), the steady-state fertility rate and level of savings were $n_p^* = \frac{(1-\tau)\gamma[\alpha+\tau(1-\alpha)]}{q\alpha\beta\pi+q(1+\gamma)[\alpha+\tau(1-\alpha)]}, s_p^* = \frac{\alpha A\beta\pi(1-\tau)(1-\alpha)k^{*\alpha}}{\alpha\beta\pi+(1+\gamma)[\alpha+\tau(1-\alpha)]}$. Also, The effect of an increase in γ, the relative preference concerning the number of children, on the steady-state level of fertility in our study and Cipriani and Fioroni (2019) are as follows: $\frac{dn^*}{d\gamma} = \frac{(1-\tau)}{q[\alpha+\tau(1-\alpha)]}\frac{\alpha+\tau(1-\alpha)+\alpha\beta\pi}{(1+\gamma+\beta\pi)^2} > 0, \frac{dn_p^*}{d\gamma} = \frac{(1-\tau)[\alpha+\tau(1-\alpha)][\alpha+\tau(1-\alpha)+\alpha\beta\pi]}{q\{\alpha\beta\pi+(1+\gamma)[\alpha+\tau(1-\alpha)]\}^2} > 0$. From $\frac{dn^*}{d\gamma}/\frac{dn_p^*}{d\gamma} = \left[\frac{(1+\gamma)[\alpha+\tau(1-\alpha)]+\alpha\beta\pi}{(1+\gamma)[\alpha+\tau(1-\alpha)]+\alpha\beta\pi+\beta\pi\tau(1-\alpha)}\right]^2 < 1$, we find that the effect of γ on the steady-state level of fertility is weaker in our study than in Cipriani and Fioroni (2019).

It is ambiguous whether p^* is larger or smaller than that in Cipriani and Fioroni (2019) where individuals maximize lifetime utility with their pension benefits as a given, (2) because k^* is smaller than that in Cipriani and Fioroni (2019), whereas n^* is higher than that in Cipriani and Fioroni (2019)s. [13]

4. Comparative Statics

This section presents the effect of changes in longevity π and contribution rate τ on the steady state under the assumption individuals maximize lifetime utility considering that their pension benefits depend on the number of their own children. In our model, longevity, and contribution rate influence individuals' savings decisions and, as a result, the long-term capital per worker, fertility rate, and pension benefits for the elderly. First, the effects of longevity on the steady-state levels of savings, capital per worker, and pension benefits per elderly are given by:

$$\frac{dk^*}{d\pi} > 0, \frac{ds^*}{d\pi} > 0, \frac{dp^*}{d\pi} < 0. \tag{20}$$

These results are identical to those of Cipriani and Fioroni (2019) where individuals make decisions with pension benefits as a given. Longevity increases relative preference for

[13] In Cipriani and Fioroni (2019), the steady-state level of pension benefits per elderly was $p_p^* = \frac{w^* \tau n^*}{\pi} = \frac{A(1-\alpha)\tau n^*}{\pi} = A(1-\alpha)\left\{\frac{Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha+\tau(1-\alpha)]}\right\}^{\frac{\alpha}{1-\alpha}} \frac{n^*}{\pi^{(1-2\alpha)/(1-\alpha)}}$.

consumption in old age. Consequently, individuals increase their savings, which in turn encourages capital accumulation. Also, as the number of elderly people receiving pension benefits increases, the pension benefits per elderly decrease. These results occur regardless of whether an individual makes decisions considering that pension benefits depend on the number of his or her children.

Next, the effect longevity on the steady-state fertility[14] is:

$$\frac{dn^*}{d\pi} = \frac{\beta(1-\tau)}{q[\alpha+\tau(1-\alpha)]}\frac{\tau(1-\alpha)-\alpha\gamma}{(1+\gamma+\beta\pi)^2} > 0 \text{ if } \tau > \frac{\alpha\gamma}{1-\alpha}. \tag{21}$$

Proposition 3: Suppose individuals maximize lifetime utility considering that their pension benefits depend on the number of their own children. Then, if pension contribution rate is sufficiently high, fertility increases with longevity in the steady state.

The effect of longevity on fertility in the steady state is ambiguous. The impacts of increasing longevity on fertility are as follows. Longevity increases individuals' relative preference for consumption in old age, which has the effect of increasing savings and reducing the number of children. This is the effect obtained in both this study and Cipriani and Fioroni (2019).

Although it does not occur in CF, there are other effects of increased longevity on fertility

14 $\frac{dn^*}{d\pi} = \frac{1-\tau}{q[\alpha+\tau(1-\alpha)]}\frac{\beta\tau(1-\alpha)(1+\gamma+\beta\pi)-\beta[\gamma\{\alpha+\tau(1-\alpha)+\beta\pi\tau(1-\alpha)\}]}{(1+\gamma+\beta\pi)^2} = \frac{\beta(1-\tau)}{q[\alpha+\tau(1-\alpha)]}\frac{\tau(1-\alpha)-\alpha\gamma}{(1+\gamma+\beta\pi)^2}$. Also, the effect longevity on the steady-state fertility in Cipriani and Fioroni (2019) is $\frac{dn_p^*}{d\pi} = -\frac{q\alpha\beta(1-\tau)\gamma[\alpha+\tau(1-\alpha)]}{\{q\alpha\beta\pi+q(1+\gamma)[\alpha+\tau(1-\alpha)]\}^2} < 0$.

in this study besides those mentioned above. In this study, individuals make decisions based on the consideration that pension benefits depend on the number of children. Therefore, increasing the number of children can mitigate the decreased pension benefits due to increased longevity. Thus, while fertility necessarily declines with increasing longevity in Cipriani and Fioroni (2019), in this study, an increase in fertility can occur with increasing longevity if the latter of the two effects described above is greater. The conditions for this are one of the following three. First, the tax rate must be sufficiently high. This is because higher pension contributions per own child also increase pension benefits. Second, the preference γ for children is sufficiently low. This is because they save more for consumption in old age, which promotes capital accumulation and increases the wage rate, leading to higher contributions to their own per-child pensions. Third is labor-intensive production. This is because when the labor share $(1-\alpha)$ is high, the wage rate is high.

Longevity increases an individual's preference for consumption in old age. In addition, an increase in the number of elderly reduces the pension benefits per elderly, p. This reduces the number of children because it decreases the lifetime income of individuals. This factor negatively affects the fertility rate. Cipriani and Fioroni (2019) demonstrated that these factors alone affect fertility. By contrast, in this study, individuals attempt to increase pension benefits, which decline with longevity, by increasing the number of their own children. This positively affects the fertility rate. This study finds a positive effect of longevity on fertility, in contrast to the negative effect obtained in Cipriani and Fioroni (2019). If the pension contribution rate is sufficiently high, having children increases the effect of increasing pension benefits, and as a result, longevity increases the fertility rate.

Finally, the effects of an increase in payroll tax on the steady-state levels of savings and capital per worker are:

$$\frac{dk^*}{d\tau} < 0 \text{ and } \frac{ds^*}{d\tau} < 0. \tag{22}$$

These results are identical to those of Cipriani and Fioroni (2019) in which individuals make decisions with pension benefits as a given. A tax increase reduces lifetime disposable income and savings and thus, capital accumulation.

5. Numerical Analysis

This section describes the simulation of the steady state and dynamics of the model. The results in the previous section are ambiguous because they depend on parameter settings. In this section, we demonstrate clear results under realistic numerical parameter settings. We compare the effects of longevity and tax increases on fertility, old-age dependency ratio, capital accumulation per worker, pension benefits, and lifetime utility, based on a numerical example from Japan, with the case in which an individual makes a decision considering that pension benefits depend on the number of their children (Case A) with the case in which an individual makes a decision with pension benefits as a given (Case B). Additionally, we compare the effect of longevity on fertility under Case A for 11 countries: Austria, Canada, France, Germany, Italy, Japan, Korea, Norway, Sweden, the United Kingdom, and the United States. The model was simulated assuming each period lasts 30 years. Therefore, the first period was up to an individual being 30 years old, the second period was 30–60 years old, and the third period was over 60 years old. The initial value of the endogenous variable and parameter settings in the model are shown in Table 1, in which round brackets indicate the year of observation. The parameters, β, and q follow those of Cipriani and Fioroni (2019). The parameters, α, π, γ, and τ are estimated from real data for each assumed country. The parameter settings for each country and the initial values of the endogenous variables employed to estimate the parameters are shown in Table 1.

		Austria	Canada	France	Germany	Italy	Japan	Korea	Latvia	Norway	Poland	Sweden	UK	US	Source
Initial value of endogenous variable	n	0.7	0.7	0.9	0.8	0.6	0.7	0.4	0.8	0.8	0.7	0.8	0.8	0.8	OECD Stat. (2021)
Parameters	α	0.325 (2012)	0.402 (2008)	0.31 (2012)	0.315 (2012)	0.313 (2012)	0.394 (2011)	0.282 (2012)	0.374 (2008)	0.454 (2012)	0.468 (2012)	0.349 (2012)	0.296 (2012)	0.363 (2011)	OECD Stat. labor share in the total economy
	β	0.3													de la Croix and Doepke (2003)
	q	0.3													Apps and Rees (2001)
	π	0.71	0.76	0.73	0.69	0.75	0.77	0.74	0.54	0.75	0.62	0.75	0.72	0.67	WHO (2019) Ratio of population aged 61-90 to 31-60
	τ	0.2	0.1	0.2	0.2	0.3	0.2	0.1	0.2	0.1	0.1	0.2	0.1	0.1	$\tau=(p/w)\times(\pi/n)$ OECD Stat. (2022)
	γ	0.35	0.34	0.52	0.43	0.3	0.36	0.13	0.44	0.41	0.34	0.44	0.39	0.4	Calculated from (17)
	A	3000													

Table1. Initial value of endogenous variable and parameter settings in the model

Source: Prepared by the author.

First, for the capital share α, we employed the labor share of OECD statistics, $1-\alpha$. For example, in the case of Japan, $1-\alpha=0.606$, so α is set to 0.394. Second, the discount factor β is $0.99^{30\times 4}=0.99^{120}=0.3$. In this model, one period lasts 30 years. Studies such as that of de la Croix and Doepke (2003) often set the quarterly utility discount factor to 0.99. In addition, the child-rearing cost q is 0.3. In the empirical literature, such as Apps and Rees (2001), the ratio of spending on children is estimated to account for 20% to 30% of a household's budget. Next, the pension contribution rate, $\tau = \frac{p_{t+1}\pi}{w_{t+1}n_t}$ is calibrated employing (2). According to the OECD statistics, the gross pension replacement rate is defined as gross pension entitlement divided by gross pre-retirement earnings, $\frac{p_{t+1}}{w_{t+1}}$, which was 34.6% in Japan in 2018. Moreover, a 53.4% old-age dependency ratio, $\frac{\pi}{n_t}$ was actualized in Japan in 2021. We calibrate the pension contribution rates using these figures, which are approximately 0.18 for Japan. In addition, the relative preference for the number of children, γ is 0.36 in Japan. Solving (15) for γ, we obtain $\gamma = \frac{(1-\tau)\beta\pi\tau(1-\alpha)-qn^*(1+\beta\pi)[\alpha+\tau(1-\alpha)]}{[qn^*-(1-\tau)][\alpha+\tau(1-\alpha)]}$. Estimating γ from (15), we obtain $\gamma = 0.36$. In 2021, the Japanese fertility rate was 1.3. Because we included single-sex individuals in the model, we set fertility equal to 0.7 per individual. The survival rate is estimated using the WHO life table; the survival rate π is the ratio of the population aged 65-104 to the population aged 30-59. Finally, the scale parameter $A = 3000$.

In the above setting, we compare the effects of increased longevity and taxes on capital accumulation per worker, fertility, savings, and pension benefits per elderly in the steady state for Cases A and B. The results of the simulation were as follows: Figures 3–7 plot the effect of longevity $\pi \in (0,1)$ in the steady state for numerical example in Japan. The solid line indicates the results of Case A, and the dotted line indicates the results of Case B.

Figure 3 plots the steady-state fertility rates for π. Comparing Cases A and B, the decrease in fertility for an increase in π is smaller in Case A than in Case B. This is because individuals seek to mitigate the effect of declining pension benefits due to increased longevity by having more children.

Figure 4 plots the steady-state old-age dependency ratio for π. Figure 5 plots the steady-state capital per worker (k^*) for π. Figure 6 plots the steady-state pension benefits (p^*) for π. Figure 7 plots the steady-state lifetime utility (U) for π. Figure 8 plots the steady-state gross pension replacement rate (p/w) for π. From Figure 5, capital accumulation per worker in Case A is lower than Case B. Figures 4, 5, 6, 7, and 8 show that the effects of longevity on the old-age dependency ratio, capital per worker, pensions benefits, lifetime utility, and gross pension replacement rate are similar in Cases A and B.

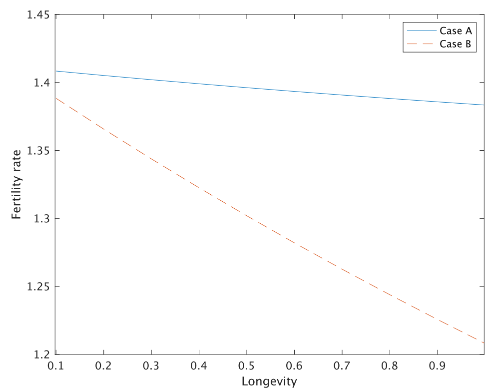

Figure 3. Fertility and longevity

Source: Prepared by the author.

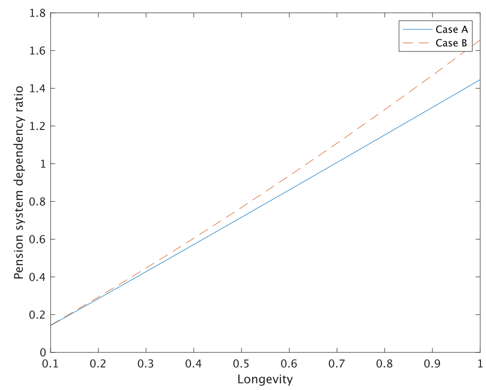

Figure 4. Old-age dependency ratio and longevity

Source: Prepared by the author.

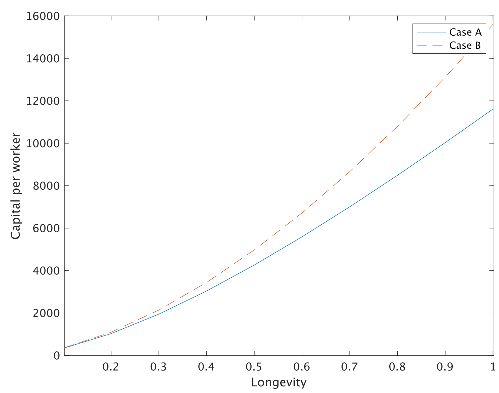

Figure 5. Capital per worker and longevity

Source: Prepared by the author.

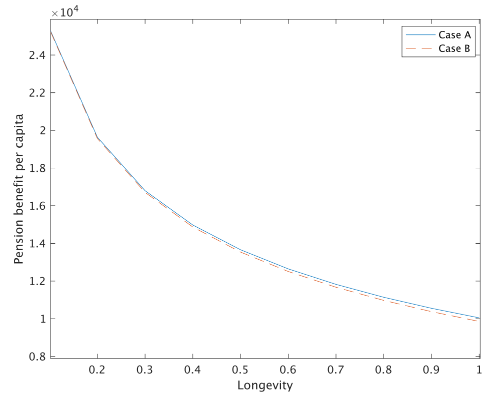

Figure 6. Pension benefits and longevity

Source: Prepared by the author.

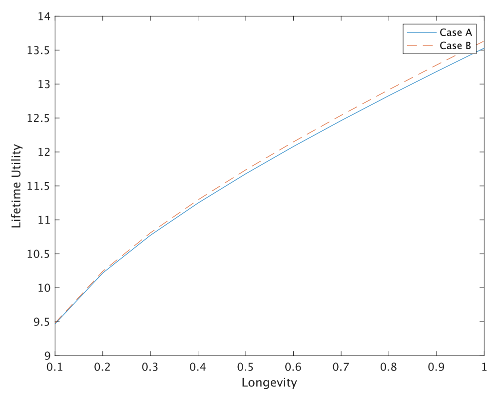

Figure 7. Lifetime utility and longevity

Source: Prepared by the author.

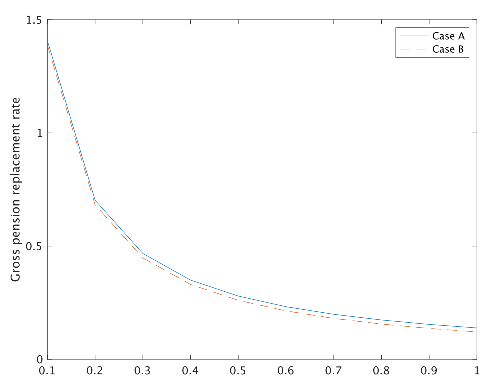

Figure 8. Gross pension replacement rate and longevity

Source: Prepared by the author.

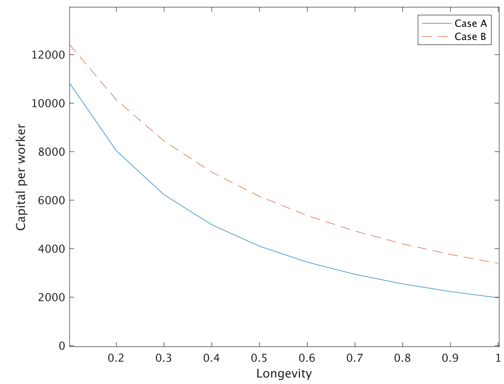

Figure 9. Capital per worker and tax rate

Source: Prepared by the author.

Figure 9 plots the steady-state capital per worker for tax rate $\boldsymbol{\tau} \in (\mathbf{0}, \mathbf{1})$ for numerical example in Japan. The solid line indicates the results of Case A; the dotted line indicates the results of Case B. The figure indicates that the effect of higher taxes on capital accumulation per worker is similar in Cases A and B.

Next, we investigate the transition paths of capital per worker, fertility rate, pension benefits, and lifetime utility due to the shock of longevity. This investigation is performed by Cipriani and Fioroni (2019). Although they have not studied fertility rates, this study examines the fertility rate transition path. We focus on the transition between the two steady states resulting from an increase in longevity from 0.57 to 0.75. We consider ten time periods, each with a duration of 30 years, and assume that longevity equals 0.57 until period 3, rises to 0.75 in period 4, and remains constant at 0.75 throughout all remaining periods. Figures 9, 10, 11, and 12 show the transition paths of capital per worker, fertility rate, pension benefits, and lifetime utility due to longevity shock, respectively for numerical example in Japan. The solid line indicates the results of Case A and the dotted line indicates the results of Case B. In Figures 10 and 13, capital accumulation and lifetime utility behave monotonically, whereas Figures 11 and 12 show that the fertility rate and pension benefits do not.

Figure 10 shows that the capital per worker increases with longevity. In both Case A and B, longevity was found to cause capital accumulation to increase. This finding is consistent with the results presented in the previous section. Nonetheless, as indicated in Proposition 1, the capital accumulation per worker in Case A is lower than that in Case B.

Figure 11 shows the transition path of the fertility rate due to longevity shock. In both Cases A and B, we found an upward spike in this rate in period 4 followed by a decrease during periods 4–5, which then stabilized from period 5 onward. The increase in period 4 is attributed to an increase in the lifetime income of individuals due to rising capital accumulation. However, longevity reduces lifetime income because it reduces pension benefits per elderly. In both cases, fertility rates up to period 4 and after period 5 were consistent with the results shown in Figure 3.

Figure 12 shows the transition path of pension benefits due to longevity shock. In both Cases A and B, we found an upward spike in pension benefits in period 5 followed by a decrease during periods 5–6, which then stabilized from period 6 onward. The increase in pension benefits in period 5 was the effect of an increase in the fertility rate. However, the fertility rate soon drops and, as a result, pension benefits decline. This result is also shown in Figure 6. In Case A, the decline in pension benefits is mitigated relative to Case B because the fertility rate is higher after period 5 than it is until period 4 in Figure 12.

Figure 13 shows the transition path of lifetime utility due to longevity shock. Lifetime utility increased with longevity in both Cases A and B. This finding is consistent with the results presented in the previous section. The lifetime utility in case A is lower than that in Case B. This is because the only factor contributing to output growth per capita in both Cases A and B are capital accumulation, and capital accumulation in Case A is lower than Case B.

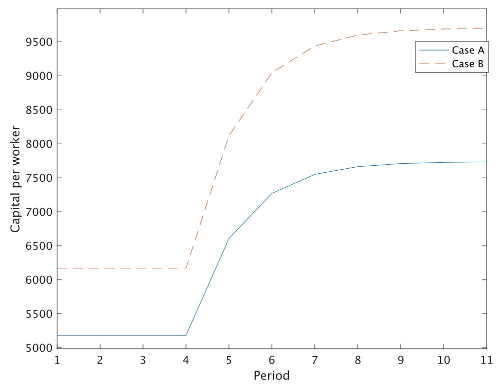

Figure 10. Capital per worker dynamics

Source: Prepared by the author.

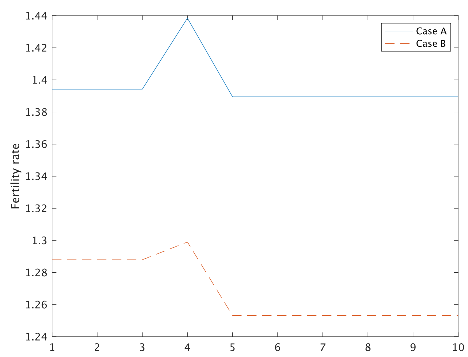

Figure11. Fertility rate dynamics

Source: Prepared by the author.

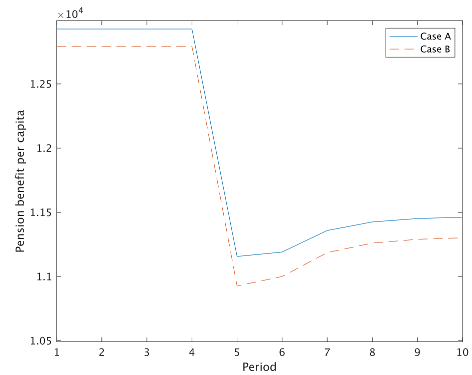

Figure 12. Pension benefits dynamics

Source: Prepared by the author.

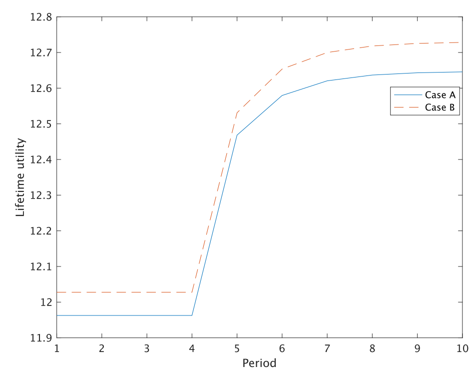

Figure 13. Lifetime utility dynamics

Source: Prepared by the author.

Finally, we performed the same numerical simulations as above, using numerical examples from major OECD countries. They are shown in Figures 14-17, with the left-hand figure showing Case A and the right-hand one showing Case B. Figure 14 plots the steady-state fertility rates for π for examples of figures from main OECD countries. Figure 15 shows the transition path of the fertility rate due to longevity shock for examples of figures from main OECD countries. Figure 16 plots the steady-state fertility rates for π for examples of figures from PAYG DC countries. Figure 17 shows the transition path of the fertility rate due to longevity shock for examples of figures from PAYG DC countries. As shown in the previous section, countries with the lowest preference concerning the number of children γ, namely Italy and South Korea, show an increase in fertility to a rise in π in Case A. In numerical examples for countries other than Italy and South Korea, the decrease in fertility for an increase in π is smaller in Case A than in Case B.

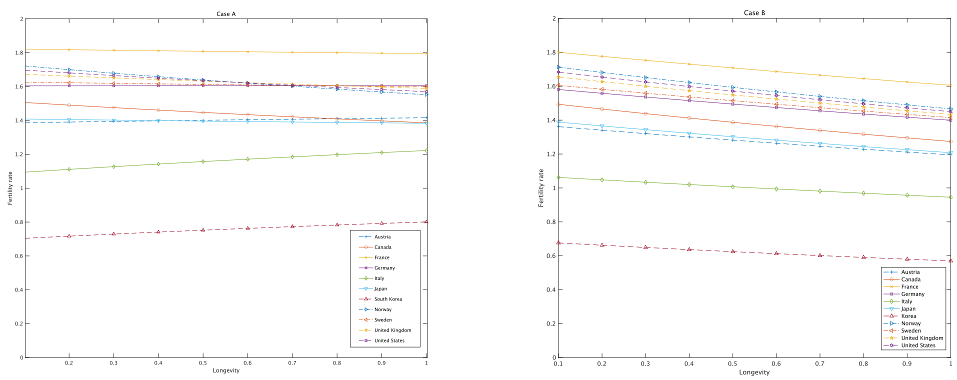

Figure 14. Fertility and longevity (main OECD Countries)

Source: Prepared by the author.

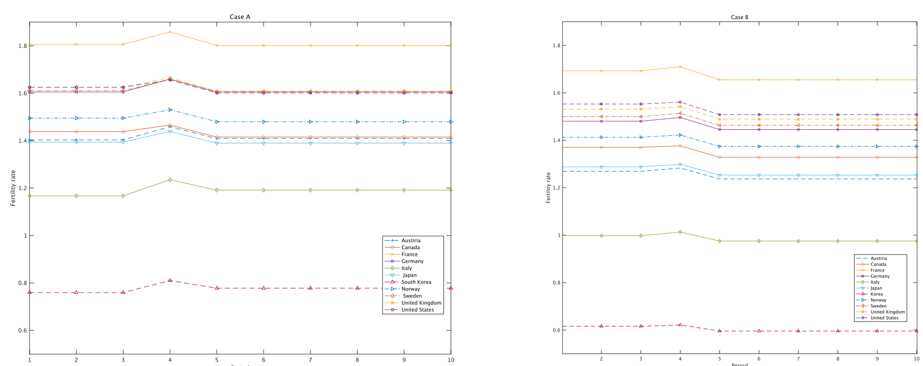

Figure 15. Fertility dynamics (main OECD Countries)

Source: Prepared by the author.

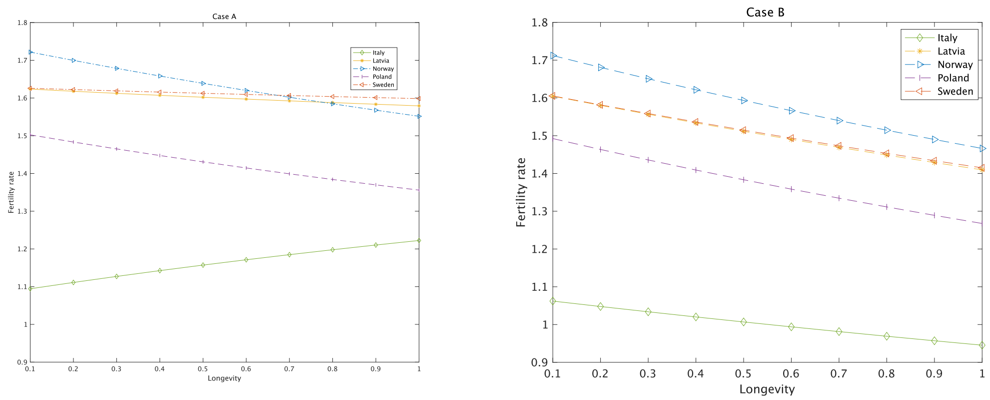

Figure 16. Fertility and longevity (PAYG DC Countries)

Source: Prepared by the author.

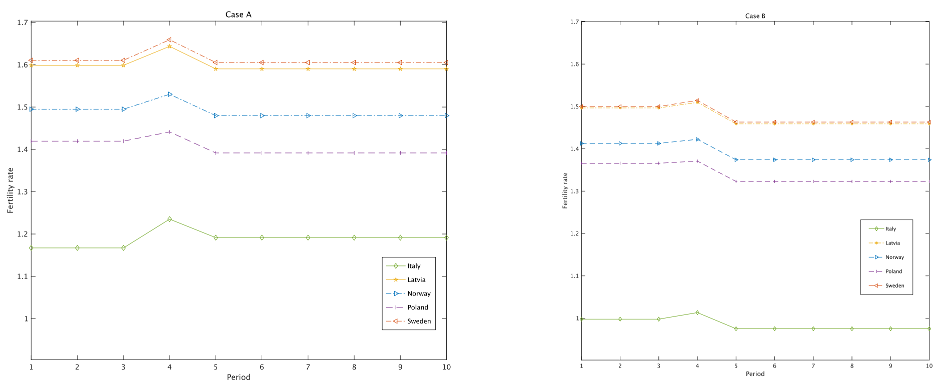

Figure 17. Fertility dynamics (PAYG DC Countries)

Source: Prepared by the author.

6. Conclusions

In this study, we examined the effects of changes in longevity and the pension contribution rate on the fertility rate and capital accumulation, employing a three-period overlapping generations model with PAYG and a defined contribution pension scheme. A feature of our model is that individuals maximize their utility by considering that their number of children affects their pension benefits. In Cipriani and Fioroni (2019), individuals maximized their utility with pension benefits as a given. The results obtained in this study were compared with those of Cipriani and Fioroni (2019) as follows. First, compared with Cipriani and Fioroni (2019), this study showed smaller capital accumulation. Second, the fertility rate was higher in the present study than that in Cipriani and Fioroni (2019). Third, longevity increases the fertility rate when the pension contributions are sufficiently high. Individuals know that having children can increase their pension benefits, which positively affects the fertility rate. This result contrasts with Cipriani and Fioroni (2019) showing that longevity decreases fertility. Our results suggest that enhancing individuals' pension knowledge and awareness can mitigate the decline in fertility rates. One means of achieving this is, for example, to strengthen pension education in schools and other institutions, as well as advertising of pension system. In addition, the government must set tax rates high enough to have the effect of increasing the fertility rate. The challenge of this study is that the only factor contributing to economic growth is the accumulation of capital per capita. In this study, the effect of capital accumulation is lower than in Cipriani and Fioroni (2019). As a result, both the economic growth rate and utility level are lower than in Cipriani and Fioroni (2019). Future research includes an examination of the factors that increase the rate of economic growth as individuals enhance their understanding of the pension system.

Appendix A : Capital Accumulation Equation per Worker

First, substituting (6) and (7) into (14) for $x = 0$, we obtain

$$k_{t+1} = \frac{s_t}{n_t} = \frac{(qw_t R^e_{t+1} - \tau w^e_{t+1})(1+\gamma+\beta\pi)}{\gamma R^e_{t+1}(1-\tau)w_t} \times \frac{(1-\tau)w_t R^e_{t+1}[qw_t R^e_{t+1}\beta\pi - \tau w^e_{t+1}(\gamma+\beta\pi)]}{R^e_{t+1}(qw_t R^e_{t+1} - \tau w^e_{t+1})(1+\gamma+\beta\pi)}$$

$$= \frac{qw_t\beta\pi - \tau\frac{w^e_{t+1}}{R^e_{t+1}}(\gamma+\beta\pi)}{\gamma} k_{t+1}. \tag{A1}$$

Next, substituting (11) and (12) into (A1), we obtain

$$k_{t+1} = \frac{qA(1-\alpha)k_t^\alpha\beta\pi - \tau\frac{1-\alpha}{\alpha}k_{t+1}(\gamma+\beta\pi)}{\gamma} k_{t+1}. \tag{A2}$$

Finally, solving (A2) for k_{t+1}, we obtain (14).

Appendix B : The Steady-State Fertility Rate and Savings Level

First, we derive the steady-state fertility rate. Substituting (11) and (12) into (6), we obtain:

$$n_t = \frac{\gamma R^e_{t+1}(1-\tau)w_t}{(qw_t R^e_{t+1} - \tau w^e_{t+1})(1+\gamma+\beta\pi)} = \frac{\gamma(1-\tau)}{\left(q - \tau\frac{w^e_{t+1}}{R^e_{t+1}}\frac{1}{w_t}\right)(1+\gamma+\beta\pi)}$$

$$= \frac{\gamma(1-\tau)}{\left(q - \tau\frac{1-\alpha}{\alpha}k_{t+1}\frac{1}{A(1-\alpha)k_t^\alpha}\right)(1+\gamma+\beta\pi)}. \tag{B1}$$

As k has one globally stable steady-state, (B1) in the steady-state is as follows:

$$n^* = \frac{\gamma\alpha A(1-\tau)}{(q\alpha A - \tau k^{*1-\alpha})(1+\gamma+\beta\pi)}. \tag{B2}$$

Substituting (16) into (B2), we obtain:

$$n^* = \frac{\gamma\alpha A(1-\tau)}{\left(q\alpha A - \tau\frac{Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha+\tau(1-\alpha)]+\beta\pi\tau(1-\alpha)}\right)(1+\gamma+\beta\pi)}$$

$$= \frac{(1-\tau)\{\gamma[\alpha+\tau(1-\alpha)]+\beta\pi\tau(1-\alpha)\}}{q(1+\gamma+\beta\pi)[\alpha+\tau(1-\alpha)]}$$

$$= \frac{(1-\tau)\{\gamma[\alpha+\tau(1-\alpha)]+\beta\pi\tau(1-\alpha)\}}{q\alpha\beta\pi+q(1+\gamma)[\alpha+\tau(1-\alpha)]+q\beta\pi\tau(1-\alpha)}. \tag{17}$$

Next, we derive the steady-state level of savings. Substituting (11) and (12) into (7), we obtain:

$$s_t = \frac{(1-\tau)w_t[qw_tR^e_{t+1}\beta\pi - \tau w^e_{t+1}(\gamma+\beta\pi)]}{(qw_tR^e_{t+1}-\tau w^e_{t+1})(1+\gamma+\beta\pi)} = \frac{(1-\tau)\left[qw_t\beta\pi - \tau\frac{w^e_{t+1}}{R^e_{t+1}}(\gamma+\beta\pi)\right]}{\left(q-\tau\frac{w^e_{t+1}}{R^e_{t+1}}\frac{1}{w_t}\right)(1+\gamma+\beta\pi)}$$

$$= \frac{(1-\tau)\left[qA(1-\alpha)k_t^\alpha\beta\pi - \tau\frac{1-\alpha}{\alpha}k_{t+1}(\gamma+\beta\pi)\right]}{\left(q-\tau\frac{1-\alpha}{\alpha}k_{t+1}\frac{1}{A(1-\alpha)k_t^\alpha}\right)(1+\gamma+\beta\pi)} \tag{B3}$$

As k has one globally stable steady-state, (B3) in the steady-state is as follows:

$$s^* = \frac{(1-\tau)A(1-\alpha)[q\alpha A\beta\pi - \tau(\gamma+\beta\pi)k^{*1-\alpha}]k^{*\alpha}}{(q\alpha A - \tau k^{*1-\alpha})(1+\gamma+\beta\pi)}. \tag{B4}$$

Substituting (16) into (B4), we obtain:

$$\begin{aligned} s^* &= \frac{(1-\tau)A(1-\alpha)}{1+\gamma+\beta\pi} \\ &\quad \times \frac{\alpha Aq\{\gamma[\alpha+\tau(1-\alpha)]\beta\pi + \beta\pi\tau(1-\alpha)\} - \tau(\gamma+\beta\pi)Aq(1-\alpha)\alpha\beta\pi}{\gamma[\alpha+\tau(1-\alpha)] + \beta\pi\tau(1-\alpha)} \\ &\quad \times \frac{\gamma[\alpha+\tau(1-\alpha)] + \beta\pi\tau(1-\alpha)}{\alpha Aq\{\gamma[\alpha+\tau(1-\alpha)] + \beta\pi\tau(1-\alpha)\} - \tau Aq(1-\alpha)\alpha\beta\pi} k^{*\alpha} \\ &= \frac{\alpha A\beta\pi(1-\tau)(1-\alpha)k^{*\alpha}}{(1+\gamma+\beta\pi)[\alpha+\tau(1-\alpha)]} = \frac{\alpha A\beta\pi(1-\tau)(1-\alpha)k^{*\alpha}}{\alpha\beta\pi + (1+\gamma)[\alpha+\tau(1-\alpha)] + \beta\pi\tau(1-\alpha)}. \end{aligned} \tag{18}$$

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