On the Sensitivity of Tests of the Mean-Variance Efficiency

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1. Introduction

This paper is concerned with the sensitivity of tests of the mean-variance efficiency of a portfolio to different sets of assets. If a new asset is added to the original set of assets,what kind of new asset would inverse the inference of the efficiency? This is the question to be investigated in this paper.

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) has been studied and tested in many papers. The theory provides an intuitively appealing relation between the return and risk of asset. That is, the expected return on any asset is positively and linearly related to the asset's beta calculated against the market portfolio. The beta is considered as the measure of risk relative to the market portfolio. Moreover,the beta is sufficient enough to describe the cross-sectional relationship between the expected return and risk. The standard tests of the CAPM are based on regression techniques with various adaptations. Usually, time-series regressions are run in the way that the return of each asset is regressed onto the return of a market index to estimate the beta of each asset, and then cross-sectional regressions are run in the way that the return is regressed onto the estimated beta. Reported are the estimates of slope in the cross-sectional regressions. Data are usually grouped to reduce measurement errors of betas. For some notable examples, see Black, Jensen and Scholes (1972), and Fama and MacBeth (1973).

Roll (1977, 1978) has raised serious doubts whether these procedures are tests of the CAPM. He concludes that the only testable hypothesis associated with the CAPM is that the market portfolio is mean-variance efficient. Roll (1977) as well as Ross (1977) have emphasized that the mean-variance efficiency of the market portfolio is mathematically equivalent to a positive linear relation between the expected return and the beta calculated against the market portfolio. Roll (1977) deduces that the CAPM "is not testable unless the exact composition of the market portfolio is known and used in the tests. This implies that the theory is not testable unless all individual assets are included in the sample." For empirical tests, market proxies have been used instead of the market portfolio, and therefore, the theory has not been tested.

While the market portfolio identification problem constitutes a severe limitation to the testability of the theory, the question of the mean-variance efficiency of a particular market proxy with respect to a subset of all assets is still an interesting problem of statistical inference. Another related question is the sensitivity of tests to different sets of assets. Since it is impossible in practice to include all existing assets for tests of the efficiency of a given portfolio, we are always confronted with the question of what happens to the inference if the efficiency is tested in a larger set of data. Stambaugh (1982), for example, has empirically investigated this question and has found that the

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addition of just a few assets to the original sample can produce a change in inference about the efficiency of a portfolio.

This paper investigates the addition of what kind of asset would likely inverse the inference in which a given portfolio is inferred as efficient with respect to the original set of assets. Therefore, it provides a theoretical explanation to empirical findings such as Stambaugh's. Section 2 presents several tests of the mean-variance efficiency of a market proxy. Then, Section 3 investigates why the inference of the efficiency is inverted by the addition of a new asset to the original sample of assets. Section 4 relates this study to previous empirical findings. Section 5 concludes this study.

2. Tests of the Mean-Variance Efficiency

Several tests of the mean-variance efficiency are briefly explained for backgrand knowlegde. Let $r_{jt} \equiv$ the return on risky asset j in period t, j = 1, 2,..., N, and t=1, 2,..., T; $r_f \equiv$ the return on the riskless asset; $r_{pt} \equiv$ the return on a market proxy whose efficiency is being tested; $z_{jt} \equiv r_{jt} - r_f \equiv$ the excess return on asset j; $z_t \equiv$ the vector of excess returns for assets; $z_{pt} \equiv r_{pt} - r_f \equiv$ the excess return on the market proxy. Also, consider the following multivariate linear regression model.

$$z_{jt} = \alpha_{pj} + \beta_{pj} z_{pt} + \varepsilon_{jt} \tag{1}$$

where ε_{jt} is the disturbance term for asset j in period t. In matrix notation, it is written as

$$z_t = \alpha_p + \beta_p z_{pt} + \varepsilon_t \tag{2}$$

where β_p is the vector of β_{pj}, α_p is the vector of α_{pj}, and ε_t is the vector of ε_{jt}, j = 1, 2,..., N. The disturbances are assumed to be jointly normally distributed each period with mean zero and nonsingular covariance matrix Σ, conditional on the excess returns of the market proxy. The disturbances are also assumed to be independent over time.

If the portfolio p is mean-variance efficient, then for the given N assets the CAPM condition must be satisfied:

$$E[z_{jt}] = \beta_{pj} E[z_{pt}] \tag{3}$$

Therefore, combining the condition in (3) with the distributional assumptions in (1) yields the following parametric restriction, which is the null hypothesis for the mean-variance efficiency of the portfolio

$$\alpha_{pj} = 0 \qquad \forall j = 1, \dots, N \tag{4}$$

or equivalently in vector notation,

$$a_p = 0l \tag{5}$$

where l is the vector of N ones.

If we estimate the system (2) using ordinarly least squares for each individual equation, the vector of estimated intercepts $\hat{a}_p$ has a multivariate normal distribution, conditional on z_{pt}. For tests of $a_p = 0l$, we define two statistics W_u and W. Statistic W_u is defined by

$$W_u \equiv \frac{\hat{a}_p' \hat{\Sigma}^{-1} \hat{a}_p}{1+\hat{\theta}_p^2} \tag{6}$$

where $\hat{\Sigma}$ is the unbiased estimator of residual covariance matrix, and $\hat{\theta}_p$ is the estimator of the Sharpe measure of the market proxy without an adjustment for degrees of freedom. The Sharpe measure of any portfolio is the slope of the line connecting the riskless rate r_f and the portfolio in the space of the mean-standard deviation of asset return. The unbiased estimator of Σ is given by

$$\hat{\Sigma} = \frac{1}{T-2} \hat{\varepsilon}'\hat{\varepsilon} \tag{7}$$

where $\hat{\varepsilon}$ is the vector of regression residuals. The estimator of the Sharpe measure θ_p is of form

$$\hat{\theta}_p = \frac{\bar{z}_p}{\hat{\sigma}_p} \tag{8}$$

where $\bar{z}_p$ is the sample mean of z_{pt}; $\hat{\sigma}_p^2 = \frac{1}{T}\Sigma\,(z_{pt} - \bar{z}_p)^2 =$ the sample variance without an adjustment for degrees of freedom. Statistic W has the same form as W_u, but based on the maximum likelihood estimator of Σ. The maximum likelihood estimator of Σ is given by

$$\hat{\Sigma} = \frac{1}{T} \hat{\varepsilon}'\hat{\varepsilon} \tag{9}$$

The only difference between W_u and W is the degrees of freedom used in the estimators of Σ, as shown in (7) and (9).

Test statistics for the mean-variance efficiency of a portfolio are monotone transforms of either W_u or W. Using W_u, Shanken's (1985) Cross-Sectional Regression (CSR) test based on Hotelling T^2 is written as

$$\mathrm{CSR} = \mathrm{T}W_u \tag{10}$$

The CSR is approximately distributed as $T^2(N-2,\ T-2)$, which converges to $\chi^2(\mathrm{N}-2)$ as $\mathrm{T} \rightarrow \infty$. For this test, see Shanken (1985) and Roll (1985). Using W_u, a F test is also constructed by

$$F=\frac{(T-N-1)\,T}{(T-2)\,N}W_u \tag{11}$$

The F has a non-central F distribution with N degrees of freedom in the numerator and $(T-N-1)$ degrees of freedom in the denominator. The non-central parameter λ of the F distribution is

$$\lambda=\frac{T}{1+\hat{\theta}_p^2}\,a'\Sigma^{-1}a \tag{12}$$

For this test, see Gibbons, Ross and Shanken (1989), and MacKinlay (1987). Using W, the likelihood ratio test and the Lagrange multiplier test are constructed. The likelihood ratio (LR) test is given by

$$LR=T\ln\,(1+W) \tag{13}$$

and the Lagrange multiplier (LM) test is given by

$$LM=\frac{TW}{1+W} \tag{14}$$

Random variables LR and LM converge in distribution to $\chi^2(N-2)$ as $T\to\infty$. Work related to the likelihood ratio test includes Gibbons (1982), Jobson and Korkie (1982), Kandel (1984), Kandel and Stambaugh (1987). Work related to comparisons of some or all of these tests includes Stambaugh (1982), Amsler and Schmidt (1985), Gibbons, Ross and Shanken (1989).

If there exist only risky assets, and the market proxy's zero covariance portfolio is used instead of the riskless asset, then the estimated disturbance vector is used in statistic W_u or W, and the degrees of freedom are adjusted accordingly.

Using a geometric approach, Gibbon, Ross and Shanken (1989) provide an intuitive interpretation of the tests for the efficiency of a portfolio. An important contribution of their paper is that they show

$$a_p'\,\Sigma^{-1}a_p=\theta_T^2-\theta_p^2 \tag{15}$$

where θ_T is the Sharpe measure of the tangent portfolio, θ_p is the Sharpe measure of the portfolio whose efficiency is tested. This holds for sample estimates as well as population parameters. Let $\hat{a}_p'\,\hat{\Sigma}^{-1}\hat{a}_p=\hat{\theta}_T^2-\hat{\theta}_T^2$ where $\hat{\Sigma}$ is the maximum likelihood estimator of Σ. Then, the statistic W can be written as

$$W=\frac{\hat{\theta}_T^2-\hat{\theta}_p^2}{1+\hat{\theta}_p^2} \tag{16}$$

The other statistic W_u has the same form as W, except the adjustment for the degrees of freedom is involved.

Therefore, one can conclude that the tests for efficiency are all based on $\hat{\theta}_T^2 - \hat{\theta}_p^2$, i. e. the difference in the squared estimators of the Share measures between the tangent portfolio and the tested portfolio. Using this interpretation, one can now investigate the sensitivity of the tests to the selection of assets.

3. Sensitivity of Tests

If new assets are added to the original set of assets, then the tangent portfolio has a higher Sharpe measure as the portfolio frontier expands. Therefore, the inference of the efficiency might be reversed even if the market proxy is inferred as efficient with respect to the original set of assets. With the inclusion of new assets, the above tests are all based on

$$\hat{\theta}_{T^*}^2 - \hat{\theta}_p^2 \tag{17}$$

where superscript* indicates the inclusion of the added assets. The difference in the squared Sharpe measures before and after the addition of assets is given by

$$[\hat{\theta}_{T^*}^2 - \hat{\theta}_p^2] - [\hat{\theta}_T^2 - \hat{\theta}_p^2] = \hat{\theta}_{T^*}^2 - \hat{\theta}_T^2 \tag{18}$$

Therefore, whether or not the addition of new assets changes the inference depends on the difference in the Sharpe measures of the tangent portfolio before and after the addition of new assets. Let z ≡ the vector of the excess returns on N original assets; $\mu \equiv E(z)$ ≡ the vector of expected excess returns on the original N assets; r_a ≡ the return on an asset (or portfolio) added to the original set of assets; $z_a \equiv r_a - r_f$ ≡ the excess return on the added asset; $\mu_a \equiv E(z_a)$ ≡ the expected excess return on the added asset; V ≡ the covariance matrix of the original N assets; σ_a^2 ≡ the variance of the return on the added asset; u ≡ the vector of covariances between the added asset and the original assets.

In matrix notation we let

$$z^* \equiv \begin{bmatrix} z \\ z_a \end{bmatrix}$$

$$\mu^* \equiv \begin{bmatrix} \mu \\ \mu_a \end{bmatrix} \tag{19}$$

$$V^* \equiv \begin{bmatrix} V & \mu \\ u' & \sigma_a^2 \end{bmatrix}$$

Then the squared Sharpe measure of the tangent portfolio with respect to the set inclusive of the added asset is given by

$$\theta_{T^*}^2 = \mu^{*'} V^{*-1} \mu^* \tag{20}$$

or equivalently

$$\theta_{T^*}^2 = \mu' V^{-1} \mu + \frac{(\mu_a - u' V^{-1} \mu)^2}{(\sigma_a^2 - u' V^{-1} u)} \tag{21}$$

The first term of the right-hand side of the equation is the squared Sharpe measure of the old tangent portfolio before the addition of the asset. That is,

$$\theta_T^2 = \mu' V^{-1} \mu \tag{22}$$

Therefore, the difference in the squared Sharpe measures of the new and old tangent portfolios is give by

$$\theta_{T^*}^2 - \theta_T^2 = \frac{(\mu_a - u' V^{-1} \mu)^2}{(\sigma_a^2 - u' V^{-1} u)} \tag{23}$$

Can a meaningful interpretation be found for the right-hand side of the equation? Fortunately, the right-hand side can be interpreted as a test statistic for the intercept term in a linear regression of z_a on z. To see this interpretation, recall that random variables z_a and z have finite second moments and their covariance matrix is denoted by

$$V^* \equiv \begin{bmatrix} V & u \\ u' & \sigma_a^2 \end{bmatrix}$$

Then one can always write

$$z_a = a + b'z + e \tag{24}$$

where $b = V^{-1} u$, $a = \mu_a - u' V^{-1} \mu$, $E(e_t) = 0$, $Var(e_t) = \sigma_a^2 - u' V^{-1} u$, and $E(z_t e_t) = 0$. Notice that $a + b' z_t$ in the equation is the best linear predictor of z_{at} given z_t because that a and b can be shown to be the value of α and β that minimize $E(z_a - \alpha - \beta' z)^2$. Therefore, the intercept term a measures how much the excess return of the added asset has a predictable component orthogonal to the excess returns of the original assets. $Var(e_t)$ shows the variation unexplained by the linear regression. Because of orthogonality, the following is satisficed

$$\| z_a \|^2 = \| a \|^2 + \| b'z \|^2 + \| e \|^2 \tag{25}$$

where $\| x \|$ is the norm of x. Therefore, the difference in the squared Sharpe measures is written as

$$\theta_{T^*}^2 - \theta_T^2 = \frac{\| a \|^2}{\| e \|^2} = \frac{a^2}{Var(e)} \tag{26}$$

This is a test statistic for the null hypothesis Ho: $a = 0$. If the excess return data of the added asset have a variation explained by the intercept term, then $\| a \|^2$ is nonzero. Additionally, if the variation explained by the intercept term is large relative to the variation unexplained by the linear regression, the ratio $\frac{\| a \|^2}{\| e \|^2}$ is large. Adding such an asset likely reverses the inference of the efficiency even if the market proxy is inferred as efficient with respect to the original set of assets. In short, the degree of the orthogonality between the excess return data of the added asset and the excess return data of the original assets is crucial to whether the inference about the efficiency would be inverted or not.

Next, this paper investigates the case in which the market proxy is efficient with respect to the original set of assets. In this case, the CAPM holds for the original assets with respect to the market proxy, and the following condition is satisfied

$$\mu_{jt} = \beta_{pj} E(z_{pt}) \tag{27}$$

or in vector notation

$$\mu_t = \beta_p E(z_{pt}) \tag{28}$$

where β_p is the vector of β_{pj}, j = 1, 2,..., N. Also, for the added asset we have

$$\mu_{at} = a + b'\mu_t \tag{29}$$

Substituting the above relation into this equation gives

$$\mu_{at} = a + b'\beta E(z_{pt}) \tag{30}$$

Using the definion $b_p \equiv b'\beta$, a linear regression of the excess return of the added asset onto the excess return of the market proxy is written as

$$z_{at} = a_p + b_p z_{pt} + e_{pt} \tag{31}$$

where $b_p = \frac{\sigma_{pa}}{\sigma_p^2} = \frac{Cov(z_{pt}, z_{jt})}{Var(z_{pt})}$, $a_p = E(z_{at}) - b_p E(z_{pt})$, $E(e_{pt}) = 0$, $Var(e_{pt}) = \sigma_a^2 - \frac{\sigma_{pa}^2}{\sigma_p^2}$, and $E(z_{pt} e_{pt}) = 0$.

Then using $\mu_t = \beta E(z_{pt})$ gives

$$a = E(z_a) - b'\beta E(z_{pt}) \tag{32}$$

Using $b_p \equiv b'\beta$ yields

$$a = E(z_a) - bE(z_{pt}) \tag{33}$$

which is equal to a_p. Therefore, the intercept term a measures how much the added asset has a predictable component orthogonal to the market proxy if the market proxy is mean-variance efficient with respect to the original set of assets. This indicates that if the added asset is highly orthogonal to the market proxy, then adding such an asset likely inverse the inference in which the market proxy is efficient with respect to the original set of assets.

Next, we consider the three situations regarding the location of the added asset in the mean-standard deviation space of asset return: (1) the added asset is located above the feasible set of original assets, (2) the added asset is within the feasible set of original assets, (3) the added asset is located below the feasible set of original assets. Here the market proxy is assumed to be efficient with respect to the original set. That is, the market proxy's Sharpe measure is the maximum with respect to the feasible set of original assets.

First, if the return of the added asset is located above the original feasible set in the mean-standard deviation space, then the following condition is met

$$\frac{E[r_a - r_f]}{\sigma_a} > \frac{E[r_p - r_f]}{\sigma_p} \tag{34}$$

where the left-hand side of the inequality is the Sharpe measure of the added asset and the right-hand side is the Sharpe measure of the market proxy. The correlation coefficient of the market proxy and the added asset is given by

$$\frac{Cov(r_p, r_a)}{\sigma_p \sigma_a} = \rho \tag{35}$$

or equivalently,

$$\frac{1}{\sigma_a} = \frac{\rho \sigma_p}{Cov(r_p, r_a)} \tag{36}$$

Substituting this into the inequality gives

$$\frac{E[r_a - r_f]\rho}{Cov(r_p, r_a)} > \frac{E[r_p - r_f]}{\sigma_p^2} \tag{37}$$

Since $\frac{\rho}{Cov(r_{pt}, r_{jt})}$ is positive,the inequality is transformed to

$$E[r_a - r_f] > \frac{1}{\rho} \frac{Cov(r_p, r_a)}{Var(r_p)} E[r_p - r_f] \tag{38}$$

Because the correlation coefficient is smaller than or equal to 1 in absolute value, the right-hand side of the inequality gets smaller and smaller as the positive correlation between r_p and r_a becomes stronger. Thus, it is impossible to have the following equality

$$E[r_a - r_f] = \frac{Cov\ (r_p, r_a)}{Var(r_p)} E[r_p - r_f] \tag{39}$$

That is, the CAPM does not hold for any asset which is located above the portfolio frontier in the mean-standard deviation space. In other words, if an asset has Sharpe measure higher than the maximum Sharpe measure with respect to the original assets, then the addition of such an asset likely inverses the inference about the efficiency of the market proxy. This is because the excess return of this kind of asset is highly orthogonal to the excess returns of the original assets.

Second, if the return of the added asset is located within the original feasible set, as opposing to the first case, the following condition is met

$$\frac{E[r_a - r_f]}{\sigma_a} \leq \frac{E[r_p - r_f]}{\sigma_p} \tag{40}$$

or equivalently

$$E[r_a - r_f] \leq \frac{1}{\rho} \frac{Cov\,(r_p, r_a)}{Var\,(r_p)} E[r_p - r_f] \tag{41}$$

The right-hand side of the inequality gets smaller and smaller as the positive correlation between r_p and r_a becomes stronger. Therefore, it is possible that the CAPM holds for this case, and the addition of such an asset does not reverse the inference in which the market proxy is inferred as efficient.

Third, if the return of the added asset is located below the original feasible set, the following condition is met

$$\frac{E[r_a - r_f]}{\sigma_a} < - \frac{E[r_p - r_f]}{\sigma_p} \tag{42}$$

or equivalently

$$E[r_a - r_f] < - \frac{1}{\rho} \frac{Cov\,(r_p, r_a)}{Var\,(r_p)} E[r_p - r_f] \tag{43}$$

The right-hand side of the inequality gets larger and larger as the positive correlation between r_a and r_p becomes stronger, and therefor it is impossible to have the CAPM relation in this case.

In short, if an added asset is located outside of the original feasible set in the mean-standard deviation space, the CAPM does not fold for such an asset. This kind of asset is highly orthogonal to the tangent portfolio with respect to the original assets, and the addition of such an asset likely inverses the inference in which the testing portfolio is inferred as efficient with respect to the original set of assets.

4. On Empirical Studies

It is worthwhile to relate this study to previous empirical findings on the sensitivity of tests of the mean-variance efficiency, especially to Stambough's finding.

Stambaugh (1982) has empirically investigated the sensitivity of tests to the composition of a market proxy and the selection of assets used in the tests. Tests are conducted with bonds, real assets, and consumer durables in addition to common stocks. He has found that inferences about the efficiency are less sensitive to the composition of the market proxy, but more sensitive to the selection of assets used in the tests. This study provides an explanation why the inferences are sensitive to the selection of assets. If the return data of added assets are highly orthogonal to the space spanned by the return data of the original assets, then the inclusion of such assets will likely inverse the inference in which the market proxy is inferred as efficient.

Kandel and Stambough (1987) have provided a sensitivity analysis to test whether a given observable market proxy is correlated at least ρ_0 with the ex ante tangent portfolio (the market portfolio) of the global universe. They have tested a joint hypothesis that (1) the unobservable benchmark portfolio is the ex ante tangent portfolio and (2) the unobservable benchmark portfolio is highly correlated with the NYSE-AMEX index at least ρ_0. This hypothesis almost always rejected for ρ_0 equal 0.9 and is often rejected for ρ_0 equal to 0.8 and even 0.7. Since their framework of sensitivity analysis is different from this study's framework, this study cannot be directly applied into their framework. However, this study may provide another way to look at their tests. This is because they have shown that the tangent portfolio T and any other portfolio p relate as follow:

$$\rho = \frac{\theta_p}{\theta_T} \tag{44}$$

For the portfolio p to be inferred as efficient, ρ needs to be close to one. Using this relation, the difference between the squared Sharpe measures of T and p is expressed as

$$\theta_T^2 - \theta_p^2 = \theta_T^2\,(1-\rho)^2 \tag{45}$$

As the correlation ρ gets smaller in positive value, the testing portfolio has a smaller Sharpe measure relative to the tangent portfolio. Therefor, it is clear that the closeness between a given portfolio and the tangent portfolio is expressed by their correlation coefficient or the difference of their squared Sharpe measures. Consequently, it seems that some assets used in their tests are highly orthogonal to the testing market index, and this refects as a relatively small correlation coefficient or a relatively large difference of the squared Sharpe measures.

5. Conclusion

This paper investigates why the inference about the mean-variance efficiency of a particular portfolio is inverted by the addition of a new asset to the original sample of assets. The inverse of inference depends on the degree of the orthogonality between the return of the added asset and the returns of the original assets. In a linear regression in which the excess return of the added asset is regressed onto the excess returns of the original assets, if the variation of the intercept term is large relative to the variation of the disturbance term, then the inclusion of such an asset likely inverses the inference in which the portfolio was inferred as efficient with respect to the original set of assets. This suggests that a portfolio is inferred as efficient if the return data of assets used for the tests lie mostly in the space spanned by the return data of the tested portfolio. Based on this analysis, an explanation can be reasoned why a particular portfolio is inferred as inefficient or efficient, or why the inference is inverted by addition of a new asset to the original sample.

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