A Simple Fixed Point Algorithm for a Static General Equilibrium Model with Tax Policies

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1. INTRODUCTION

A purely theoretical representation of an economy in the Walrasian general equilibrium framework was formalized in the 1950s. Ever since the outset of its theoretical formulation, the general equilibrium framework has been very intractable in dealing with multi-dimensional empirical issues due to the lack of both efficiently operational algorithms and computational power. However, this kind of criticism no longer holds because both recent technical advancements in computer technology and subsequent refinements in operational algorithms have been made since the first applications of Scarf's algorithm (1973).

In this paper, we applied a simple fixed point algorithm to the general equilibrium model to demonstrate that the algorithm can efficiently execute a structurally more complex general equilibrium model with various tax policy options than the model used in our earlier paper (Tanaka and Kawano, 1996). We believe that this algorithm can be of considerable empirical use. We followed the solution procedure set out by Shoven and Whalley (1984, 1992) who considerably simplified the general equilibrium solution procedure by elegantly reducing the dimensionality of the solution space to the number of factors of production in the Walrasian general equilibrium structure. The main feature of this algorithm is its relative simplicity in both concept and programming, in comparison with the unit simplex methods, such as the algorithms of Scarf (1973) and others briefly described by Shoven and Whalley (1992), for a general equilibrium model.

We demonstrated the empirical usefulness of this algorithm in the framework of the same two-good-two-factor general equilibrium model incorporating a system of equal tax yield termed as *differential tax incidence* by Musgrave (1959) and its related government expenditure[1]. With the presence of taxation, we experimented with the competitive equilibria under the nine tax policy options. Satisfactory results were gained and presented in tables 1- 7 with the computational time ranging from 0.033 to 0.150 seconds for each tax policy simulation.

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1) The methods for incorporating taxes into a formal general equilibrium model were shown by Shoven and Whalley (1973), Ballard, Fullerton, Shoven, and Walley (1985).

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2. THE OUTLINE OF THE MODEL

2.1. *The demand side of the model*

There are two consumers, one rich (R) and the other poor (P), in the economy.

Both consumers maximize their own utilities by solving the following constrained maximization :

$$Max_{[X_1^m,\, X_2^m \geq 0]}\; U^m(X_1^m, X_2^m)$$

$$s.t.\ \sum_{i=1}^{2} p_i(1+\tau_i)X_i^m = Y_0^m - t_y^m(Y_0^m - \mathrm{F}) \qquad (1)$$

$$\because\ Y_0^m = wL^m + rK^m + \gamma^m T$$

where

X_i^m = ith commodity demand ($i=1,2$) for the mth consumer ($m= R, P$),

L^m = labor endowments for the mth consumer ($m= R, P$),

K^m = capital endowments for the mth consumer ($m= R, P$),

P_i = producer price of commodity i ($i=1, 2$),

$P_i\,(1+\tau_i)$ = consumer's price ($i=1, 2$),

τ_i = commodity tax rate ($i=1, 2$),

τ_l^i = payroll tax rate (the labor tax rate could also vary by industry) ($i=1, 2$),

$w(1+\tau_l^i)$ = corresponding user price of labor for producers ($i=1, 2$),

τ_k^i = sector-specific capital income taxes ($i=1, 2$),

$r(1+\tau_k^i)$ = gross-of-tax price of capital in sector i ($i=1, 2$),

τ_y = (marginal) income tax rate,

F = real personal exemption,

$\tau_y\,(Y^m - F)$ = tax liability of the mth household ($m= R, P$),

γ^m = revenue-distribution shares among consumers ($m= R, P$),

T = tax revenues,

$\gamma^m T$ = revenue transfer for mth household ($m= R, P$),

w = wage rate,

r = rental rate,

T = tax revenues,

γ^m = revenue-distribution shares among consumers ($m=R, P$),

Y_0^m = given income for the mth consumer ($m=R, P$)

$= wL^m + rK^m + \gamma^m T$,

$U^m(.)$ = a well behaved neoclassical homothetic (strictly quasiconcave) utility function for the mth consumer ($m=R$, P)[2].

The commodity demands for two consumers are the solutions to problem (1). The demand function for mth consumer[3] ($m=R$, P) is :

$$X_i^m = X_i^m(p_1(1+\tau_1), p_2(1+\tau_2), Y_0^m - t_y^m(Y_0^m \text{ -F})) \\ = X_i^m(p_1(1+\tau_1), p_2(1+\tau_2), r, w, T) \tag{2}$$

2.2. *The production side of the model*

There are two industries ($i=1, 2$). We assume aggregate (constant returns to scale[4]) industry production functions for both industries. An aggregate producer in each perfectly competitive industry[5] maximizes his profit by solving the following constrained cost-minimization problem :

$$Min_{[L^i, K^i \geq 0]}\ C^i(L^i, K^i) = w(1+\tau_l^i)L^i + r(1+\tau_k^i)K^i \\ s.t.\quad Q^i(L^i, K^i) = Q_0^i,\quad \forall\ (i=1, 2) \tag{3}$$

where

C^i = (direct) cost function,

L^i = labor demand,

K^i = capital demand,

Q_0^i = given output level,

$Q^i(.)$ = A well behaved neoclassical (strictly quasiconcave) production function for the ith aggregate industry production function ($i=1, 2$),

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2) If individuals in each group have both identical and homethetic preferences, a higher utilty of each group indifference curve indicates the potential welfare improvement for all individuals in a group.

3) If the values of w, r, and T change, then the income of the mth consumer ($m=R, P$) is expressed as $Y^m(r, w, T)=wL^m+rK^m+\gamma^m T$. The resulting demand functions X_i^m are now assumed to be homogeneous of degree zero in (r, w, T). Since govemment revenue T is a part of the demand function, the problem of simultaneity that consumers face in determining their demands can be solved(Ballard, Fullerton, Shoven, and Whalley, 1985).

4) At the competitive equilibrium price with zero profits, the firms can operate at any level of their production and market share. The assumption of a constant returns to scale technology in industry structure seems appropriate in the long run in this general equilibrium framework. Varian (p.356,1992) holds his view that "if we can always replicate, the only sensible long run technology is a constant-returns-to scale technology."

5) Let us quote the phrase of Varian (p. 216,1992):
The question is not whether any particular market is perfectly competitive-almost no market is. The appropriate queation is to what degree models of perfect competition can generate insights about real-world markets.

τ_l^i = payroll tax rate (the labor tax rate could also vary by industry) ($i=1, 2$),
$w(1+\tau_l^i)$ = corresponding user price of labor for producers ($i=1, 2$),
τ_k^i = sector-specific capital income taxes ($i=1, 2$),
$r(1+\tau_k^i)$ = gross-of-tax price of capital in sector i ($i=1, 2$).

The derived factor (labor and capital) demand functions for given output level Q_0^i are :

$$L^i = L^i(r(1+\tau_k^i),\ w(1+\tau_l^i),\ Q_0^i),$$
$$K^i = K^i(r(1+\tau_k^i),\ w(1+\tau_l^i),\ Q_0^i). \quad (4)$$

Therefore, the factor demand functions as output changes are given as :

$$L^i = L^i(Q^i | r(1+\tau_k^i),\ w(1+\tau_l^i)),$$
$$K^i = K^i(Q^i | r(1+\tau_k^i),\ w(1+\tau_l^i)). \quad (5)$$

2.3. *Excess demand conditions*

Excess demand conditions[6] for goods, factors, and tax revenues[7] are:

$$\sum_{m=R}^{P} X_i^m(p_1(1+\tau_1),\ p_2(1+\tau_2),\ r,\ w,\ T) - Q^i \leq 0,$$
$$\sum_{i=1}^{2} L^i(Q^i \mid r(1+\tau_k^i),\ w(1+\tau_l^i)) - \sum_{m=R}^{P} L^m \leq 0,$$
$$\sum_{i=1}^{2} K^i(Q^i \mid r(1+\tau_k^i),\ w(1+\tau_l^i)) - \sum_{m=R}^{P} K^m \leq 0,$$
$$\sum_{i=1}^{2} \sum_{m=R}^{P} \tau_i p_i X_i^m(r,\ w,\ T)$$
$$+ \tau_l \sum_{i=1}^{2} wL^i(r(1+\tau_k^i),\ w(1+\tau_l^i),\ T) \quad (6)$$
$$+ \sum_{i=1}^{2} \tau_k^i r K^i(r(1+\tau_k^i),\ w(1+\tau_l^i),\ T)$$
$$+ \sum_{m=R}^{P} \tau_y(wL^m + rK^m + \gamma^m T - F) - T \leq 0$$
$$\because \quad (i = 1, 2), \quad (m=R,\ P).$$

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6) The existence and uniqueness of competitive equilibrium are discused in Arrow and Hahn (1971), and elsewhere. In the case of a general equilibrium framework with taxes, see an existence proof in Shoven and Whalley (1973).

7) In this model, a govemment budget imbalance equation must be included.

2.4. *Zero-profit conditions*

If the output of industry i is positive, the price of output i is equal to the long run average costs (zero-profit in the long run) under perfect competition.

$$p_i(r(1+\tau^i_k),\ w(1+\tau^i_l)) = \frac{C^i(L^i\,(.),\ K^i(.))}{Q^i}$$

$$= \frac{w(1+\tau^i_l)L^i(Q^i \mid r(1+\tau^i_k),\ w(1+\tau^i_l)) + r(1+\tau^i_k)K^i(Q^i \mid r(1+\tau^i_k), w(1+\tau^i_l),)}{Q^i},$$

$$= w(1+\tau^i_l)l^i\,(1 \mid r(1+\tau^i_k),\ w(1+\tau^i_l)) + r(1+\tau'^i_k)\,k^i(1 \mid\ r(1+\tau^i_k),\ w(1+\tau^i_l)), \tag{7}$$

$$where\ \ \frac{L^i}{Q^i} = l^i\,(1 \mid r(1+\tau^i_k),\ w(1+\tau^i_l)),$$

$$\frac{K^i}{Q^j} = k^i\,(1 \mid r(1+\tau^i_k),\ w(1+\tau^i_l)).$$

$$\forall (i=1,\ 2)$$

2.5. *Walras's law*

Finally, any set of prices for a general equilibrium model must satisfy *Walras's law* for theoretical consistency. Walras's law in this model is stated as:

$$\begin{aligned}
&\sum_{i=1}^{2} p_i(1+\tau_i)\left[Q^i - \sum_{m=R}^{P} X_i^m(.)\right] + \\
&w(1+\tau'^i_l)\left[\sum_{i=1}^{2} L^i\,(.)\ - \sum_{m=R}^{P} L^m\right] + \\
&r(1+\tau^i_k)\ \left[\sum_{i=1}^{2} K^i(.)\ - \sum_{m=R}^{P} K^m\right] + \\
&\sum_{i=1}^{2}\sum_{m=R}^{P} \tau_i p_i X_i^m\,(.) + \sum_{i=1}^{2}\tau^i_l wL^i\,(.) + \\
&\sum_{i=1}^{2}\tau^i_k rK^i\,(.) + \sum_{m=R}^{P}\tau_y(wL^m + rK^m + \gamma^m T - F) - T = 0.
\end{aligned} \tag{8}$$

3. A SOLUTION PROCEDURE

The solution procedure for the general equilibrium model can be considerably simplified by reducing the dimensionality of the solution space to the number of factors of production. In other words, the equilibrium characterized by a vector $(p_1,\ p_2,\ r,\ w,\ T)$ for this two-good-two-factor model with various tax policy options is characterized by a vector $(r^*,\ w^*,\ T^*)$. We follow the Shoven and Whalley procedure[8] set out as :

Step 1: Determine the factor demands per unit of output i, since both factor demand functions L^i and K^i are derived as solutions to constrained cost-minimization problem (3).

$$
\begin{aligned}
\frac{L^i}{Q^i} &= l^i[r(1+\tau_k^i),\ w\ (1+\tau_l^i),\ 1], \\
&= l^i(r,\ w,\ 1) \\
\frac{K^i}{Q^i} &= k^i[r(1+\tau_k^i),\ w\ (1+\tau_l^i),\ 1], \\
&= k^i(r,\ w,\ 1) \\
&\qquad \forall (i=1,\ 2)
\end{aligned}
\tag{9}
$$

Step 2: Compute the commodity price P_i with the zero-profit conditions that the price of output i is equal to the long run average cost (s).

$$
\begin{aligned}
p_i(r,\ w) &= w(1+\tau_l^i)l^i(.)+r(1+\tau_l^i)k^i(.), \\
&\qquad \forall (i=1,\ 2)
\end{aligned}
\tag{10}
$$

Step 3: Compute the individual commodity demands $(X_1^R,\ X_2^R,\ X_1^P,\ X_2^P)$ by substituting the commodity prices (P_1 and P_2) computed in step 2.

$$
\begin{aligned}
X_i^m(r,\ w,\ T) &= X_i^m[(1+\tau_1)p_1(.),\ (1+\tau_2)p_2(.),\ r,\ w,\ \tau_y^m,\ F,\ \gamma^m T] \\
&\qquad \forall (i=1,\ 2,\ m=R,\ P)
\end{aligned}
\tag{11}
$$

Step 4: Compute the market demands for the two commodities by the two consumers, and then compute the outputs of both commodities through the market clearing condition for the two commodity markets.

$$
\begin{aligned}
Q^i(r,\ w) &= \sum_{m=R}^{P} X_i^m(r,\ w,\ T),\quad (i=1,\ 2), \\
&\equiv Z^i(r,\ w,\ T)
\end{aligned}
\tag{12}
$$

8) See chapter 2 in Shoven and Whalley (1992).

Step 5 : Compute the factor demand functions L^i and K^i through (9).

$$L^i(r,w,T)=l^i(r,w,1)*Z^i(r,w,T)$$
$$K^i(r,w,T)=k^i(r,w,1)*Z^i(r,w,T) \tag{13}$$
$$\forall(i=1,2)$$

Step 6 : Find the converged equilibrium values r^* and T^* in both excess factor demand functions, ρ_k for capital K and ρ_l for labor L, given initial values for the variable parameters r and T by treating the wage rate w as a numéraire. When the converged r^* and T^* are found, both excess factor demand functions and the government budget imbalance equation ρ_k, ρ_l, and ρ_G simultaneously converge to zeros[9]. An equilibrium is characterized by a vector (r^*, w^*, T^*), since both demand and supply functions are now homogeneous of degree zero in factor prices and tax revenue.

$$\begin{aligned}
\rho_k(r,T|w=1) &= \sum_{i=1}^{2}K^i(r,w,T)-\sum_{m=R}^{P}K^m,\\
\rho_l(r,T|w=1) &= \sum_{i=1}^{2}L^i(r,w,T)-\sum_{m=R}^{P}L^m,\\
\rho_G(r,T|w=1) &= \sum_{i=1}^{2}\sum_{m=R}^{P}\tau_i p_i X_i^m(r,w,T)+\sum_{i=1}^{2}\tau_l^i wL^i(r,w,T)\\
&+\sum_{i=1}^{2}\tau_k^i rK^i(r,w,T)+\sum_{m=R}^{P}\tau_y(wL^m+rK^m+\gamma^m T-F)-T.
\end{aligned} \tag{14}$$

4. STEPS IN APPLYING THE ALGORITHM TO THE MODEL

The purpose of applying this algorithm to our general equilibrium model is to demonstrate that the algorithm can efficiently execute a structurally more complex general equilibrium model with various tax policy options than the model used in our earlier paper (Tanaka and Kawano, 1996). We attempt to find the converged equilibrium values r^* and T^* in both excess factor demand functions ρ_k , ρ_l and in the government budget imbalance equation ρ_G. We treat the wage rate w as a numéraire. When the simultaneously converged equilibrium values r^* and T^* are found for both excess factor demand functions ρ_k and ρ_l, the government budget imbalance equation ρ_G must also converge to zero due to *Walras's*

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9) *Walras's law* also guarantees that the value of the sum of the two excess factor demand functions ρ_k and ρ_l is zero, thus the remaining govemment budget imbalance equation ρ_G must be zero.

law. Therefore, we can concentrate on working on the functions ρ_k and ρ_l so as to find the converged equilibrium values r^* and T^*. The steps involved in applying this algorithm are stated as follows :

Step 1: Give a set of initial values (r_1, T_1) for the set of parameters (r, T), and calculate the corresponding starting values $\rho_k\ (r_1, T_1 | w=1)$ and $\rho_l\ (r_1, T_1 | w=1)$ through steps 1 to 6 in the general equilibrium solution procedure in section 3. Then, assign the arbitrary incremental step lengths Δ_r and Δ_T.

Step 2: Now hold T fixed at the arbitrarily chosen level as shown in Figure 1, and then calculate the first sequential sets[10)] of ρ_k and ρ_l towards the first target function $\rho_k\ (r, T | w=1)=0$ by changing the parameter value of r with the predetermined Δ_r parallel to the horizontal r-axis in the same manner as step 3 in our earlier paper (1996)[11)]. Repeat this sequence of calculations until the case of step 3-a (see footnote 11) emerges, that is, the sign of ρ_k changes.

Step 3: In the same way, hold r fixed, instead of T, at the chosen level as shown in step 2 in Figure 1, and then calculate the second sequential sets of ρ_k and ρ_l towards the second target function $\rho_l\ (r, T | w=1) = 0$ by changing the parameter value of T by the predetermined Δ_T parallel to the vertical T-axis. Repeat this sequence of calculations until the case of step 3-a (see footnote 11) emerges, that is, the sign of ρ_l changes.

Step 4: Repeat steps 2 through 3 in the cobweb form as illustrated in Figure 1 for both excess factor demand functions $\rho_k\ (r, T | w=1)$ and $\rho_l\ (r, T | w=1)$ by changing the target functions alternately until both ρ_k and ρ_l converge to zeros simultaneously so as to find the converged equilibrium values r^* and T^*. The remaining government budget imbalance equation ρ_G must also be balanced at the converged equilibrium level of the parameter values r^* and T^*.

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10) In the same manner as step 2 in our earlier paper (1996), the first sequential set of ρ_k and ρ_l is $\{\rho_k(r_2=r_1+\Delta_r,\ T_2=T_1 | w=1),\ \rho_l(r_2=r_1+\Delta_r,\ T_2=T_1 | w=1)\}$,where the endpoint r_2,of each step length is constantly renamed as the beginning point r_1 of the next step length as we move along towards the first target function $\rho_k(r, T | w=1) = 0$ in Figure 1.

11) Step 3 in our earlier paper is briefly summarized as follows:

Step 3-a: If the signs of $\rho_k(r_1, T_1 | w=1)$ and $\rho_k(r_2, T_2 | w=1)$ are different, reduce the currently assigned incremental value Δ_r by half. Otherwise, proceed to step 3-b.

Step 3-b: If $| \rho_k(r_1, T_1 | w=1) | < | \rho_k(r_2, T_2 | w=1) |$, reverse the direction of the incremental value Δ_r. Otherwise, proceed to step 3-c.

Step 3-c: If $| \rho_k(r_1, T_1 | w=1) | > | \rho_k(r_2, T_2 | w=1) |$, reassign r_2, T_2, and the corresponding value $\rho_k(r_2, T_2 | w=1)$ (*which have already been calculated in step 2*) as r_1, T_1, and $\rho_k(r_1, T_1 | w=1)$ respectively.

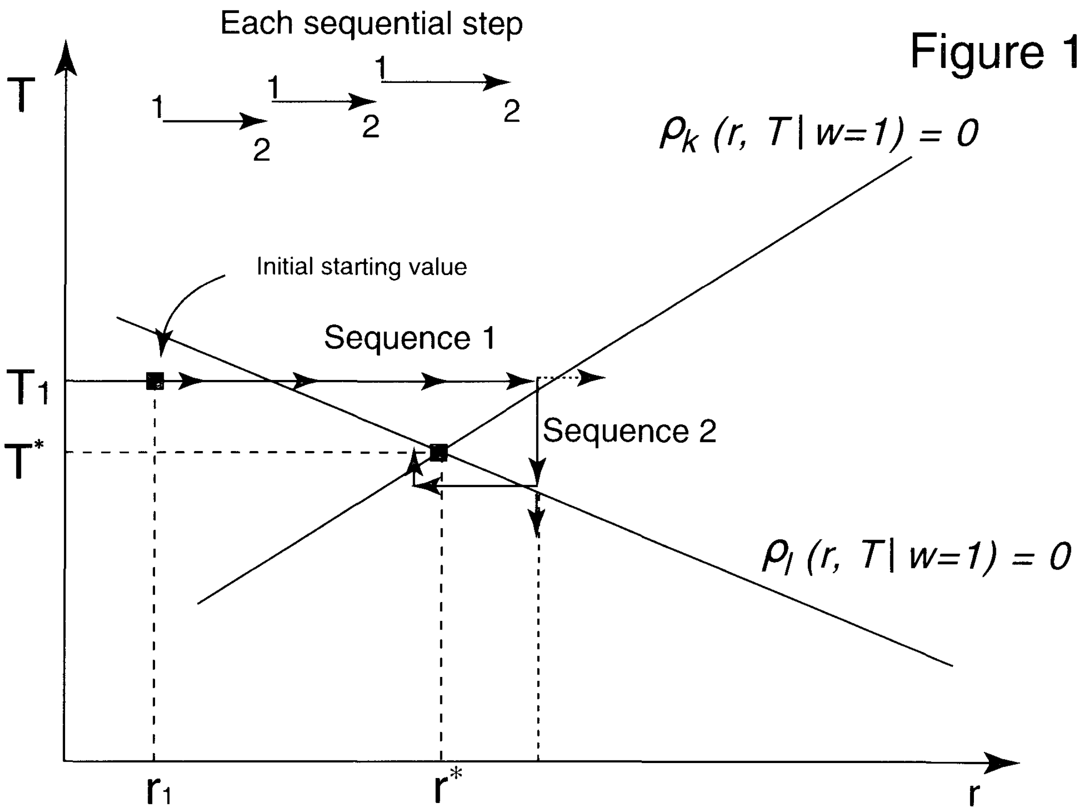

Figure 1

5. SPECIFICATION OF PARAMETER VALUES AND FUNCTIONAL FORMS

5.1 *Parameter values*

Parameter values for this simple general equilibrium economy are specified in the following tables:

Demand side & factor endowments

α_1^R	α_2^R	α_1^P	α_2^P	μ^R	μ^P	K^R	K^P	L^R	L^P
0.5	0.5	0.3	0.7	1.5	0.75	25	0	0	60

where

α = consumer share parameters between x_1 and x_2.

μ = elasticities of substitution in consumption between x_1 and x_2

L^m = household labor endowments $(m = R, P)$.

K^m = household capital endowments $(m = R, P)$.

Supply side

ψ^2	ψ^2	σ^1	σ^2	δ^1	δ^2
1.5	2.0	0.6	0.7	2.0	0.5

where

ψ = parameters for scale factors.

δ = factor weighing parameters.

σ = elasticities of substitution between factor inputs K^i and L^i.

5.2 *Functional forms*

The functional forms for this simple general equilibrium economy with both CES utility and production functions are specified, and the resulting output and factor demands are as follows:

Demand side

CES utility functions :

$$U^m = \left(\sum_{i=1}^{2} (\alpha_i^m)^{1/\mu^m} (x_i^m)^{(\mu^m-1)/\mu^m} \right)^{\mu^m/(\mu^m-1)}$$

$$\forall \ m=R, P.$$

Demand functions :

$$x_i^m = \frac{\alpha_i^m [Y^m - \tau_y^m (Y^m - F)]}{[(1+\tau_i)p_i]^{\mu^m} \left(\sum_{i=1}^{2} \alpha_i^m [(1+\tau_i)p_i]^{(1-\mu^m)} \right)}$$

$$\because \quad Y^m = wL^m + rK^m + \gamma^m T$$

$$\forall \ (m=R, P, \ i=1, 2)$$

Supply side

CES production functions :

$$Q^i = \Psi^i \left(\delta^i (L^i)^{(\sigma^i-1)/\sigma^i} + (1-\delta^i)(K^i)^{(\sigma^i-1)/\sigma^i} \right)^{\sigma^i/(\sigma^i-1)}$$

$$\forall \ i=1,2.$$

Factor demand functions :

$$L^i=\frac{1}{\Psi^i}\,Q^i\left[\delta^i+(1-\delta^i)\left(\frac{\delta^i r(1+\tau_k^i)}{(1-\delta^i)w(1+\tau_l^i)}\right)^{(1-\sigma^i)}\right]^{\sigma^i/(1-\sigma^i)}$$

$$K^i=\frac{1}{\Psi^i}\,Q^i\left[\delta^i\left(\frac{(1-\delta^i)w(1+\tau_l^i)}{\delta^i r(1+\tau_k^i)}\right)^{(1-\sigma^i)}+(1-\delta^i)\right]^{\sigma^i/(1-\sigma^i)}$$

$$\forall(i=1,2),\ (m=1,2)$$

6. NUMERICAL SIMULATIONS FOR TAX POLICY OPTIONS

We have demonstrated that our algorithm can efficiently execute the model by way of nine numerical examples for tax policy options listed in the table below. Policy 1 indicates a no policy intervention equilibrium. Policies 2-9 indicate the case of an equal-tax-yield equilibrium termed as *differential tax incidence* by Musgrave (1959). The satisfactory results are presented in the subsequent tables 1-7. In this model, the competitive equilibria must be examined with care. Prices are sellers' prices for inputs (prices net-of-input taxes) and wholesale prices for outputs (prices net-of-consumer taxes). The same number of dollar revenue yield ($34.710 in our examples) for each tax regime does not mean the same thing, since the amount of government purchases are influenced by different price levels[12]. Finally, the computational time for each policy in table 2 ranges 0.033 to 0.150 seconds. Our algorithm seems to execute the model with great efficiency and accuracy.

Tax policy options

Policy 1	No policy intervention
Policy 2	Imposed values: *Payroll tax rate:* $\tau_l^1=\tau_l^2=0.3$, *Commodity tax rate:* $\tau_1=0.2$, $\tau_2=0.1$, *Revenue share:* $\gamma^R=0.4$, $\gamma^P=0.6$, *No other taxes.* Results: *Tax revenue: T=34.710.*
Policy 3	Imposed values: *Tax revenue: T=34.710, Revenue share:* $\gamma^R=0.4$, $\gamma^P=0.6$, *No other taxes.* Results: *Commodity tax rate:* $\tau_1=0.547$, $\tau_2=0.273$.
Policy 4	Imposed values : *Tax revenue : T=34.710, Commodity tax rate :* $\tau_1=0.6$, $\tau_2=0.2$, *Revenue share:* $\gamma^R=0.4$, $\gamma^P=0.6$, *No other taxes.* Results: *Payroll tax rate:* $\tau_l^1=\tau_l^2=0.040$.
Policy 5	Imposed values: *Tax revenue: T=34.710, Revenue share:* $\gamma^R=0.4$, $\gamma^P=0.6$, *No other taxes.* Results: *Payroll tax rate:* $\tau_l^1=\tau_l^2=0.579$.

12) The detailed discussion of equal-tax-yield equilibria are provided by Ballard, Fullerton, Shoven, and Whalley (1985).

Policy 6	Imposed values: *Tax revenue: T=34.710, Revenue share:* γ^R*=0.4,* γ^P*=0.6, No other taxes.* Results: Income tax rate: $\tau_y^R = \tau_y^P = 0.269$.
Policy 7	Imposed values: *Tax revenue: T=34.710, Capital gain tax rate :* $\tau_k^1=0.5$, $\tau_k^2=0.2$, *Revenue share:* γ^R*=0.4,* γ^P*=0.6, No other taxes.* Results: Income tax rate: $\tau_y^R = \tau_y^P = 0.234$.
Policy 8	Imposed values: Tax revenue: *T=34.710, Tax exemption: F=10. Revenue share:* γ^R*=0.4,* γ^P*=0.6, No other taxes.* Results: Income tax rate: $\tau_y^R = \tau_y^P = 0.319$.
Policy 9	Imposed values: Tax revenue: *T=34.710, Revenue share:* γ^R*=0.6,* γ^P*=0.4, No other taxes.* Results: Income tax rate: $\tau_y^R = \tau_y^P = 0.270$.

Table 1. Tax policy options

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(1.1) Tax revenue	T	0	34.710	34.710*	34.710*	34.710*	34.710*	34.710*	34.710*	34.710*
(1.2) Income tax rate for rich	$\tau_y^R = \tau_y$	0	0	0	0	0	0.269	0.234	0.319	0.270
(1.3) Income tax rate for poor	$\tau_y^R = a\,\tau_y$	0	0	0	0	0	0.269 *(a=1)*	0.234 *(a=1)*	0.319 *(a=1)*	0.270 *(a=1)*
(1.4) Payroll tax rate in sector 1 (labor income tax rate)	$\tau_l^1 = \tau_l$	0	0.3*	0	0.040	0.579	0	0	0	0
(1.5) Payroll tax rate in sector 2 (labor income tax rate)	$\tau_l^2 = b\,\tau_l$	0	0.3*	0	0.040 *(b=1)*	0.579 *(b=1)*	0	0	0	0
(1.6) Capital gain tax rate for commodity 1	$\tau_k^1 = t_k$	0	0	0	0	0	0	0.5*	0	0
(1.7) Capital gain tax rate for commodity 2	$\tau_k^2 = ct_k$	0	0	0	0	0	0	0.2*	0	0
(1.8) Commodity tax rate for commodity 1	$\tau_1 = \tau$	0	0.2*	0.547	0.6*	0	0	0	0	0
(1.9) Commodity tax rate for commodity 2	$\tau_2 = d\tau$	0	0.1*	0.273 *(b=1/2)*	0.2*	0	0	0	0	0
(1.10) Tax exemption	F	0	0	0	0	0	0	0	10*	0
(1.11) Revenue distribution share for rich	γ^R	0	0.4*	0.4*	0.4*	0.4*	0.4*	0.4*	0.4*	0.6*
(1.12) Revenue distribution share for poor	γ^P	0	0.6*	0.6*	0.6*	0.6*	0.6*	0.6*	0.6*	0.4*

Note: The asterisk "*" indicates an imposed value in each policy analysis.

Table 2. The computational time and the number of iterations.

	policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(2.1) Computational time	0.033sec	0.033sec	0.150sec	0.117sec	0.017sec	0.034sec	0.05sec	0.117sec	0.033sec
(2.2) Numbers of iterations	108	102	429	411	102	141	128	422	110

Table 3. Welfare (utility) of the rich, and the poor.

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(3.1) Utility of rich household	U^R	27.872	32.055	28.536	28.973	34.825	28.597	24.882	29.238	32.633
	Rank	6	3	8	5	1	7	9	4	2
(3.2) Utility of poor household	U^P	50.891	46.447	49.853	48.992	43.623	50.133	53.865	49.463	45.915
	Rank	2	7	4	6	9	3	1	5	8

Table 4. Capital market

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(4.1) Rental rate	$r=(r/w)$	1.373	1.806	1.429	1.516	2.143	1.372	1.023	1.370	1.362
(4.2) Capital endowments (Total capital supply)	ΣK^i $(i=1,2)$	25.000	25.000	25.000	25.000	25.000	25.000	25.000	25.000	25.000
(4.3) Amount of capital used for commodity 1	K^1	6.212	5.901	5.176	4.723	6.556	6.247	4.915	6.279	6.447
(4.4) Amount of capital used for commodity 2	K^2	18.788	19.099	19.824	20.277	18.444	18.753	20.085	18.721	18.553

Table 5. Labor market

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(5.1) Wage rate	w	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(5.2) Labor endowments (Total labor supply)	ΣL^i $(i=1,2)$	60.000	60.000	60.000	60.000	60.000	60.000	60.000	60.000	60.000
(5.3) Amount of labor used for commodity 1	L^1	26.366	25.617	23.797	22.597	27.176	26.451	26.014	26.526	26.922
(5.4) Amount of labor used for commodity 2	L^2	33.634	34.383	36.206	37.403	32.834	33.549	33.986	33.474	33.078

Table 6. Commodity 1 market

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(6.1)Producer price of commodity 1 & cost per unit of output	$p_1 = C_1/Q^1$	1.399	1.824	1.413	1.476	2.202	1.399	1.436	1.399	1.396
(6.2) Total commodity 1 demanded & supplied	$X_1 = X_1^R + X_1^P = Q^1$	24.942	24.102	22.084	20.774	25.859	25.038	23.368	25.123	25.571
(6.3) Commodity 1 demanded by rich household	X_1^R	11.515	12.369	10.128	9.542	14.362	11.812	9.686	12.075	13.465
(6.4) Commodity 1 demanded by poor household	X_1^P	13.428	11.733	11.956	11.232	11.497	13.226	13.682	13.048	12.105

Table 7. Commodity 2 market

		policy 1	policy 2	policy 3	policy 4	policy 5	policy 6	policy 7	policy 8	policy 9
(7.1) Producer price of commodity 2 & cost per unit of output	$p_2 = C_2/Q^2$	1.093	1.428	1.112	1.167	1.717	1.092	1.042	1.092	1.090
(7.2) Total commodity 2 demanded & supplied	$X_2 = X_2^R + X_2^P = Q^2$	53.378	55.453	58.024	59.683	53.203	54.256	56.286	54.147	53.574
(7.3) Commodity 2 demanded by rich household	X_2^R	16.675	20.344	19.406	20.898	20.868	17.111	15.674	17.497	19.544
(7.4) Commodity 2 demanded by poor household	X_2^P	37.704	35.109	38.617	38.785	32.336	37.144	40.612	36.650	34.029

7. CONCLUSION

We applied a simple fixed point algorithm to the general equilibrium model to demonstrate that the algorithm can efficiently execute a structurally more complex general equilibrium model with various tax policy options than the model used in our earlier paper (Tanaka and Kawano, 1996). We followed the solution procedure set out by Shoven and Whalley (1984, 1992) who considerably simplified the general equilibrium solution procedure by elegantly reducing the dimensionality of the solution space to the number of factors of production in the Walrasian general equilibrium structure. In their solution procedure, the zero profit conditions of perfect competition and the linear homogeneous properties of both utility and production functions are essential by construction.

The competitive Walrasian general equilibrium structure is theoretically consistent and very useful for empirical analyses. Although the contemporary economy is depicted as "monopolistic" in many trade literatures, all agents in a whole economy in the aggregate sense cannot be monopolistic price-makers. The force of market price mechanism cannot be undermined. This Walrasian structure with the Armington assumption (Armington 1969) seems to provide an ideal starting framework for simulating and evaluating the effects of many important policy changes on resource allocation, especially in a large international economy where scale economies may be fully exploited (Whalley 1985). If scale economies and imperfect competition are incorporated into the model of a small open economy (Harris 1984), the effects of policy changes seem considerably overestimated. Our algorithm illustrated in this simple Walrasian general equilibrium framework is of considerable operational use for many empirical analyses for both closed and open economies.

Finally, we briefly state the main strengths and the possible weakness of the algorithm. First, the main strengths are (1) simplicity in both concept and programming, (2) ease of calculations (*since it does not require either simplex or Jacobian calculations*), (3) increased accuracy, and (4) assured convergence of the required parameter values with the conditions of uniqueness and stability. Second, the possible weakness is that the number of iterations for the convergence of parameter values is relatively increased. However, this weakness no longer poses a serious problem owing to the recent advancement of computer technologies.

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